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Portfolio rebalancing versus buy-and-hold: A simulation based study with special consideration of portfolio concentration

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Abstract

The aim of this study is not only to explore if portfolio rebalancing can lead to a better performance compared to a buy-and-hold (B&H) strategy but to find out if there is a correlation between the weight-based concentration of the B&H portfolio and the success of a rebalancing strategy. For these reasons, it is firstly discussed how rebalancing affects portfolio diversification, risk-adjusted return and the utility value for a certain investor. Secondly, it is discussed on what the portfolio weight of a special stock is depending on whereas the cases of an initially equally and unequally weighted portfolio are distinguished. The latter one has a larger weight concentration which is determined by the normalized Herfindahl index and the coefficient of variation. These issues are explored theoretically and empirically. In the empirical analysis the Monte Carlo simulation is used which is based upon 1,000 simulations with 520 generated returns for each of the 15 assumed stocks in the initially equally weighted portfolio. The results show that the diversification ratio, the return to risk ratio, and the utility value of the rebalanced portfolio turn out to be significantly greater than those of the B&H portfolio. The rebalanced portfolio has a slightly (not significant) positive rebalancing return. Finally, a strong negative correlation between the rebalancing return and the weight concentration of the B&H portfolio is found.

JEL classification numbers: G11
Keywords: portfolio rebalancing, rebalancing return, buy-and-hold, diversification ratio, return to risk ratio, utility value, portfolio concentration, autocorrelation

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1 Introduction

Portfolio rebalancing is the process of buying and selling portions of assets in a portfolio in order to maintain the originally determined weightings. Such a strategy calls for selling assets with a rising portfolio weight due to price changes and purchasing stocks whose portfolio weights have been reduced ("buy low and sell high"). Thus, a positive effect can be achieved for the portfolio return (Hayley et al., 2015, pp. 1, 16, 22). However, there are also critics of rebalancing who argue that a buy-and-hold (B&H) strategy might produce higher returns because this approach “lets winners run”. As it involves a onetime portfolio allocation at the beginning of the investment period with no further adjustment up to the end of the period portfolio weights will vary as a result of price changes. Thus, rising stocks automatically get a higher weight compared to falling stocks. A B&H strategy might be successful in bull phases of the stock market cycle. But a long term bull phase cannot be expected in reality which was shown by several stock market crashes in the past. So it can be profitable to sell a winning position before its downturn (Dayanandan and Lam, 2015, p.81).

The essential study of Perold and Sharpe (1988) shows that rebalancing of a portfolio to its target allocation can lead to an additional performance benefit when there is a strong mean-reverting behavior (p. 21). Further studies found that there is no guarantee for a better performance of a rebalancing strategy compared to a B&H strategy. It has been discussed in several studies to what extent rebalancing is successful. Both theoretically and empirically, the results are different and to some extent contradictory.

Tsai (2001) analyzes four commonly used rebalancing strategies. Her study evaluates portfolios that are composed of seven asset classes. She finds that the four strategies produce similar risks, returns and Sharpe ratios whereas “neglecting rebalancing produces the lowest Sharpe ratios across a wide range of risk profiles” (p. 110). Therefore, she concludes that portfolios should be periodically rebalanced.

Zilbering, Jaconetti and Kinniry (2010) find that there is no universally optimal rebalancing strategy. According to their study there are no meaningful differences “whether a portfolio is rebalanced monthly, quarterly, or annually” (p. 12).

Jones and Stine (2010) compare two rebalancing strategies with the B&H portfolio in terms of terminal wealth, risk and expected utility. They find that the measure used to rank each strategy determines the optimal strategy (p. 418).

Bouchey et al. (2012) call the extra growth that can be generated from the systematic diversification and rebalancing of a portfolio “volatility harvesting”. Focussing on equal weighting, they recommend simply diversifying and rebalancing as it enhances returns in the long term. They conclude that their advice applies to any set of volatile and uncorrelated assets that are sufficiently liquid. Therefore, they don't distinguish between mean-reverting and assets that follow a trend.

Rulik (2013) found that portfolio rebalancing does not always generate positive return. The payoff is rather depending on certain conditions. He concludes that "the rebalancing effect grows when stock volatility rises, the correlation among stocks decreases and there is less difference in stocks' returns over the long run" (p. 7). The rebalancing bonus for equal-weight portfolios was different in the examined markets. While it was positive and consistent in the U.S. market, it was almost absent for a portfolio of European stocks. The reason was the lower average correlation among the U.S. stocks.

Chambers and Zdanowicz (2014) find that "portfolio rebalancing tends to increase the expected value of a portfolio when asset prices are mean-reverting" (p. 74). They conclude that the added expected portfolio value can be attributed neither to reduced volatility nor to increased diversification.

Dichtl, Drobetz and Wambach (2014) use history-based simulations to examine whether different classes of rebalancing (periodic, threshold, and range balancing) outperform a B&H strategy. To measure the risk-adjusted performance they use the Sharpe ratio, Sortino ratio, and Omega measure. They find that the economic relevance of the choice of a specific rebalancing strategy is minor.

Hallerbach (2014) decomposes the difference between the growth rate of a rebalanced portfolio and the B&H portfolio (which is the return from rebalancing) into the volatility return and the dispersion discount. He finds that, depending on the circumstances, the rebalancing return can be positive or negative, and concludes that rebalancing cannot serve as a general "volatility harvesting" strategy. If a rebalanced portfolio consists of assets with comparable growth rates, the volatility return is likely to dominate the dispersion discount (pp. 313-314).

In a more recent paper, Meyer-Bullerdiek (2017) examined a portfolio of 15 German stocks for different rebalancing frequencies and different periods. He found that there are no clear results as the rebalancing returns can be both positive and negative. After removing five stocks from the original portfolio whose final weights (based on the total period of 520 weeks) were either relatively high or relatively low, the rebalancing return improved significantly. The revised B&H portfolio of the 10 stocks left was not as much concentrated as the original portfolio. Obviously, there should be a certain relationship between the rebalancing return and the concentration of the B&H portfolio.

Mier (2015) gives an overview over studies that have examined the performance of concentrated portfolios versus diversified portfolios. Brands, Brown and Gallagher (2005) find a positive relationship between fund performance and portfolio concentration for their sample of active equity funds. They use a divergence index developed by Kacperczyk, Sialm and Zheng (2005) as an industry concentration measure. These authors show in their study that this measure has a high correlation with the Herfindahl index and can be thought of as a market adjusted Herfindahl index (Kacperczyk, Sialm and Zheng, 2005, p. 1987). Brands, Brown and Gallagher (2005) also found that "the performance/concentration relationship is also significant (insignificant) for stocks in which managers hold overweight (underweight) positions" (p. 170).

Baks, Busse and Green (2006) analysed mutual fund performance based on four portfolio weight inequality measures: the Herfindahl Index, the normalized Herfindahl Index, the Gini coefficient, and the coefficient of variation. The authors find that "the four measures provide qualitatively similar rankings across groups of funds, with some notable differences" (p. 7). They conclude that "concentrated fund managers outperform their diversified counterparts. This result lends support to the notion that the managers who are confident in their ability assess correctly the relative merits of stocks overall as well as within their portfolios" (pp. 19-20).

Sohn, Kim and Shin (2011) use several portfolio concentration and performance measures and show that diversified funds generate better performance than focused funds. They also identify "that the underperformance of focused funds could be due to liquidity problems, idiosyncratic risk, and trading performance" (p. 135).

Yeung et al. (2012) created concentrated portfolios and showed that the absolute returns from the concentrated portfolios were higher than those from the diversified funds. The performance was even better the higher the concentration (p. 10-11). However, in their conclusion they issue the caveat "that a good diversifier will always beat a bad concentrator and that success for the investors will always come back to identifying the managers skilled at stock selection" (p. 23).

Chen and Lai (2015) use the Herfindahl index, the normalized Herfindahl index and the coefficient of variation to measure the concentration level of mutual fund holdings. They find in their study of the Taiwan equity mutual fund market that fund holdings' concentration levels are high and positively related to funds' risk-adjusted returns in tranquil market periods, but this went to the opposite in turmoil markets where risk-adjusted returns of high concentrated funds were lower than those of broadly diversified funds (pp. 284-285).

None of these studies has investigated the relationship between the success of rebalancing a portfolio versus the concentration of the corresponding B&H portfolio. Therefore, this paper will explore the difference between a rebalancing and a B&H strategy with special regard to portfolio concentration. The objective of the study is to determine whether there actually is a relationship between the weight-based concentration of the B&H portfolio and the success of a rebalancing strategy. As Hayley et al. (2015, p. 14) point out that an increase in expected terminal wealth only occurs if there is rebalancing and negative autocorrelation in relative asset returns, the aspect of autocorrelations of returns is also included in this study. The success of a rebalancing strategy will also be assessed with regard to portfolio diversification, risk-adjusted portfolio returns and utility value. These aspects will be examined for the case of independent, normally distributed equity returns. For this reason, the analysis uses a Monte Carlo simulation, which assumes normally distributed equity returns. Using a Monte Carlo simulation can avoid problems with data specific results that can arise in empirical studies (Jones and Stine, 2010, p. 406).

This paper is structured as follows: Section 2 discusses how rebalancing affects portfolio diversification, risk-adjusted return and the utility value for a certain investor – each compared to the B&H portfolio. Section 3 provides the relationship between return and the portfolio weight of a certain stock. Furthermore, following Baks, Busse and Green (2006) and Chen and Lai (2015), three statistics are presented to measure portfolio concentration associated with the portfolio weights: the Herfindahl index, normalized Herfindahl index and coefficient of variation. This section also discusses the relationship between the weight concentration of the B&H portfolio and the rebalancing return as well as the relationship between the autocorrelation of stock returns and the rebalancing return. The empirical results of the Monte Carlo simulation of a 15 stocks portfolio over 520 rebalancing periods are presented in section 4. Section 5 summarizes the main results of the study.

2 The effect of rebalancing on portfolio diversification, risk-adjusted return and utility value

This section discusses how rebalancing affects portfolio diversification, risk-adjusted return and the utility value for a certain investor – each compared to the B&H portfolio. To measure the portfolio diversification, Choueifaty and Coignard (2008, p. 41) recommended the “diversification ratio” which is defined as the ratio of the weighted average of assets’ volatilities divided by the portfolio volatility:

$$\text{Diversification ratio} = \frac{\sum_{i=1}^{n} w_i \times \sigma_i}{\sigma_p} \tag{1}$$

In this formula, w_i is the portfolio weight of asset i, σ_i is the standard deviation of asset returns and σ_p is the standard deviation of portfolio returns.

Choueifaty, Froidure and Reynier (2013, p. 2) find that "this measure embodies the very nature of diversification, whereby the volatility of a long-only portfolio of assets is less than or equal to the weighted sum of the assets' volatilities."

Assuming that there are no short-selling opportunities ("long-only"), DR will be greater or equal 1 if at least one investment in the portfolio has a positive standard deviation σ_i. In the extreme case, that all correlations between the shares were 1, the numerator and denominator of the diversification ratio would be identical. In all other cases – due to the diversification effect – the denominator is lower than the numerator. Accordingly, the diversification ratio measures the diversification performance of investments that are not perfectly correlated. In the numerator, therefore, the portfolio risk stands for the case without diversification and the denominator is the (actual) risk including diversification (Lee, 2011, p. 15-16).

In the empirical analysis in section 4 of this paper, the average (weekly) weights ($\bar{w}_i$) are used in the numerator because in this study weekly returns are assumed. Thus, equation (1) changes to:

$$\text{Diversification ratio} = \frac{\sum_{i=1}^{n} \bar{w}_i \times \sigma_i}{\sigma_p} \qquad (2)$$

To what extent a rebalancing strategy results in a better diversification for a portfolio compared to a B&H strategy can be determined by an empirical analysis of the differences between the respective diversification ratios.

The success of a rebalancing strategy shall also be assessed in terms of the risk-adjusted portfolio return. For this purpose, the so-called the return to risk ratio can be used, which quantifies the average portfolio return ($\bar{r}_p$) per unit of risk. The risk is defined as the standard deviation of the portfolio returns (σ_p). Thus, this performance measure is based on the total risk, i.e. on non-systematic and systematic risk (market risk). This makes sense if the portfolio is sufficiently diversified so that there are hardly any non-systematic risks (Culp and Mensink, 1999, p. 62).

$$\text{Return to risk ratio} = \frac{\bar{r}_p}{\sigma_p} \qquad (3)$$

The extent to which the risk adjusted performance of a rebalancing and a B&H

portfolio differs, is determined by the empirical analysis in section 4. In principle, it can be assumed that the risk of a rebalanced portfolio will be smaller, because higher concentrations in the portfolio will be avoided. However, the return can also be reduced, so that in theory hardly any statement can be made regarding the success of a rebalancing strategy with regard to the return to risk ratio. According to Dayanandan and Lam (2015, p. 89), "the virtue of portfolio rebalancing is one of the controversial issues in portfolio management. Proponents argue for it on the grounds that it de-risks the portfolio and brings value to investors. On the other hand, the critics of portfolio rebalancing argue against it both theoretically and empirically".

Finally the relationship between rebalancing and the utility value for a certain investor shall be explored. The ultimate goal for investors is actually not to maximize or minimize the performance components return and risk, but to maximize their benefits. It is assumed that investors can assign a utility score to different investment portfolios based upon risk and return. A popular function that is used by both financial theorists and practitioners assigns a portfolio the following utility score (Bodie, Kane and Marcus, 2012, p. 163):

$$U_p = E(r_p) - \frac{1}{2} \cdot A \cdot \sigma_p^2 \tag{4}$$

where U_P is the utility value of the portfolio, $E(r_p)$ is the expected portfolio return, A is an index of the investor's risk aversion, and σ_p^2 is the variance of the portfolio returns. This equation illustrates that a portfolio receives a higher (lower) utility score for a higher (lower) expected return and a lower (higher) volatility. Besides, the risk aversion is important as it "plays a large role in way investors allocate their money to various assets and also in how they revise those allocations over time" (Jones and Stine, 2010, p. 408). In section 4 of this study the utility scores of the rebalanced portfolio and the B&H portfolio are compared for different degrees of risk aversion.

3 Rebalancing Return, weight concentration and autocorrelation of returns

Following Hallerbach (2014), the rebalancing return can be described as the full difference between the geometric mean returns of a rebalanced and a B&H portfolio. He posits that the rebalancing return is composed of the volatility return and a dispersion discount. It can be expressed as follows:

$$RR_H = \bar{r}_p^g - \bar{r}_{B\&H}^g = \underbrace{\left(\bar{r}_p^g - \sum_{i=1}^{n} w_{i0} \times \bar{r}_i^g\right)}_{\text{Volatility return}} - \underbrace{\left(\bar{r}_{B\&H}^g - \sum_{i=1}^{n} w_{i0} \times \bar{r}_i^g\right)}_{\text{Dispersion discount}} \qquad (5)$$

where $\bar{r}_p^g$ is the geometric mean return of the portfolio (which is rebalanced), $\bar{r}_{B\&H}^g$ is the geometric mean return of the B&H portfolio, and w_{i0} are the initial fixed weights of the assets. Thus, the volatility return contributes positively and the dispersion discount contributes negatively to the rebalancing return. As rebalancing a portfolio means to sell assets that have outperformed the portfolio and buying assets that have underperformed ("buy low and sell high"), a larger asset volatility leads to a higher volatility return. This can be shown by the so-called diversification return for a rebalanced portfolio derived by Willenbrock (2011):

$$DR_W = \bar{r}_p^g - \sum_{i=1}^{n} w_i \times \bar{r}_i^g \approx \frac{1}{2} \times \sum_{i=1}^{n} w_i \times \left(\sigma_i^2 - \mathrm{Cov}\left(r_i, r_p\right)\right) \qquad (6)$$

where DR_W is the diversification ratio according to Willenbrock and $\mathrm{Cov}(r_i,r_p)$ is the covariance between the returns of asset i and the portfolio. This diversification return is driven by the volatility of the assets in the portfolio because of the "buy low and sell high"-strategy. Therefore, Willenbrock recommends the name "volatility return" instead of "diversification return" (p. 44).

The effect of dispersion in individual assets' geometric returns on the B&H portfolio's geometric return is reflected by the dispersion discount. Hallerbach points out that "when individual growth rates differ and time passes by, the security with the highest growth rate tends to dominate a B&H portfolio and lift its growth rate over the securities' average growth rate" (p. 302). Hence, the rebalancing return can be positive or negative dependent on the size of the dispersion discount.

In order to find a relationship between the return of a stock and its weight in the portfolio, consider a stock i with a market value of $V_{i,t}$ at the beginning of period t. The portfolio weight of the stock can be calculated as follows:

$$w_{i,t} = \frac{V_{i,t}}{V_{p,t}} \qquad (7)$$

where $V_{p,t}$ is the market value of portfolio p. The market values of the portfolio and the stock at the end of the period t (i.e. in period t+1) will be according to Hallerbach (2014, p. 302-303):

$$V_{p,t+1} = V_{p,t} \times (1 + r_{p,t}) \quad \text{and} \quad V_{i,t+1} = V_{i,t} \times (1 + r_{i,t}) \tag{8}$$

where $r_{p,t}$ is the portfolio return and $r_{i,t}$ is the asset return in period t. Therefore, at the beginning of period t+1 the stock weight will be:

$$w_{i,t+1} = w_{i,t} \times \frac{1 + r_{i,t}}{1 + r_{p,t}} \tag{9}$$

The weights of the stocks will change over period t if the stock returns differ from the portfolio returns.

Equation (9) can be rearranged as follows:

$$r_{p,t} = w_{i,t} \times \frac{1 + r_{i,t}}{w_{i,t+1}} - 1 \tag{10}$$

In case of an equally weighted portfolio at the beginning of period t, the portfolio return can be calculated as follows:

$$r_{p,t} = w_{i,t} \times r_{i,t} + (1 - w_{i,t}) \times \bar{r}_{\text{all other n}-1\text{ stocks,t}} \tag{11}$$

where $\bar{r}_{\text{all other n}-1\text{stocks,t}}$ is the arithmetic average return of all the other n-1 stocks in the portfolio at period t. In this case, $w_{i,t}$ = 1/n because the weights of all assets in the portfolio are the same:

$$r_{p,t} = \frac{1}{n} \times r_{i,t} + \left(1 - \frac{1}{n}\right) \times \bar{r}_{\text{all other n}-1\text{ stocks,t}} \tag{12}$$

If $r_{i,t} > \bar{r}_{\text{all other n}-1\text{ stocks,t}}$, then $r_{i,t} > r_{p,t}$, and according to equation (9) $w_{i,t+1} > w_{i,t}$. Therefore, in this case, a higher than average weight of a stock at the beginning of period t will be even higher in the next period if there is no rebalancing. If all stocks in the portfolio have the same return ($\bar{r}_{\text{all other n}-1\text{ stocks,t}} = r_{i,t} = r_{p,t}$), then $w_{i,t+1} = w_{i,t}$.

Plugging equation (12) into equation (9) leads to the following relationship in case of an equally weighted portfolio at the beginning of period t:

$$w_{i,t+1} = \frac{1}{n} \times \frac{1 + r_{i,t}}{1 + \frac{1}{n} \times r_{i,t} + \left(1 - \frac{1}{n}\right) \times \bar{r}_{\text{all other } n-1 \text{ stocks}, t}} \quad (13a)$$

$$w_{i,t+1} = \frac{1 + r_{i,t}}{n + r_{i,t} + (n-1) \times \bar{r}_{\text{all other } n-1 \text{ stocks}, t}} \quad (13b)$$

This equation is valid in case of an equally weighted portfolio at the beginning of period t. Hence, the weight in period t+1 is not dependent on the initial weight at period t, but on the return of the stock, the average return of the other stocks, and on the number of stocks in the portfolio. This applies to a portfolio that is rebalanced to equal weights after each period. On the other hand, a B&H portfolio will lead to different weights in period t+1 and it can be assumed that these weight differences will increase in the following periods.

If weights are constant over time, the arithmetic average return of the portfolio ($\bar{r}_p$) can be expressed as follows:

$$\bar{r}_p = \sum_{i=1}^{n} w_i \times \bar{r}_i \qquad \text{for } w_i = \text{constant} \quad (14)$$

where $\bar{r}_i$ is the arithmetic average return of stock i over all considered periods. Willenbrock (2011, p. 42) points out that this equation applies only to a rebalanced portfolio where the portfolio is rebalanced to the constant proportions at the end of each holding period.

If there are no equal weights at the beginning of period t, equation (11) is only an approximation and therefore, $r_{p,t}$ has to be calculated using the weights of all stocks in the portfolio:

$$r_{p,t} = \sum_{i=1}^{n} w_{i,t} \times r_{i,t} \quad (15)$$

Hence, in this case the weight of stock i at the beginning of period t+1 can be expressed in the following way:

$$w_{i,t+1} = w_{i,t} \times \frac{1 + r_{i,t}}{1 + r_{p,t}} = w_{i,t} \times \frac{1 + r_{i,t}}{1 + \sum_{i=1}^{n} w_{i,t} \times r_{i,t}} \quad (16)$$

This equation shows that the weight in period t+1 of a special stock in the portfolio is dependent on the initial weights of all stocks at the beginning of period t and on the returns of all stocks in period t. This applies to a portfolio that is not rebalanced to equal weights after each period. Equation (16) also shows that the weight of stock i increases (decreases) if $r_{i,t}$ is larger (smaller) than $r_{p,t}$. In case of an increasing weight, rebalancing a portfolio means to sell a certain number of stock i until the initial weight is achieved. On the other hand, if in this case stock i is part of a B&H portfolio, it will start with a higher weight into the next period.

The following example is intended to provide a better understanding of the context and calculations. Given are two portfolios that consist both of the same 5 stocks A, B, C, D, and E. The data for these stocks is presented in Table 1.

Table 1: Example – Equal weights at the beginning of t

Stock	**A**	**B**	**C**	**D**	**E**	**Total**
Portfolio weight at t	20%	20%	20%	20%	20%	**100%**
Return in period t	25%	-10%	-20%	15%	35%	
Value at t+1	25%	18%	16%	23%	27%	**109%**
Portfolio weight at t+1	22.94%	16.51%	14.68%	21.10%	24.77%	**100%**

Focussing on stock A (as stock i), according to the data of the table, the following values can be obtained:

$$\bar{r}_{\text{all other n−1 stocks},t} = \frac{-0.10-0.20+0.15+0.35}{4} = 5\%$$

$$r_{p,t} = \frac{1}{5}\times 0.25 + \frac{4}{5}\times\frac{-0.10-0.20+0.15+0.35}{4} = 9\% \quad \text{(see equation 12)}$$

$$w_{A,t+1} = \frac{(1+0.25)}{5+0.25+4\times 0.05} = 0.2294 \quad \text{(see equation 13b)}$$

Stock A's weight increases because its return is higher than the return of the entire portfolio.

It is assumed that in the next period (t+1) the returns of all 5 stocks are still the same (Scenario 1). Thus, the rebalanced portfolio return will be again 9% (as shown in Table 1). On the other hand, a B&H strategy will lead to the results presented in Table 2. It should be noted that all decimal places of the results in Table 1 (not just the two shown in the table) were included in the stock weights.

Table 2: Example: B&H portfolio at the beginning of t+1 – Scenario 1

Stock	**A**	**B**	**C**	**D**	**E**	**Total**
Portfolio weight at t+1	22.94%	16.51%	14.68%	21.10%	24.77%	**100%**
Return in period t+1	25%	-10%	-20%	15%	35%	
Value at t+2	28.67%	14.86%	11.74%	24.27%	33.44%	**112.98%**
Portfolio weight at t+2	25.38%	13.15%	10.39%	21.48%	29.60%	**100%**

According to equation (15), the portfolio return in t+1 equals 12.98% and equation (16) gives the portfolio weights at t+2. As the return of stock A is larger than the portfolio return, its portfolio weight at the beginning of t+2 is higher than one period before. A portfolio rebalanced to equal weights leads to a return of 9% in period t+1. Therefore, the B&H portfolio outperforms the rebalanced portfolio in this example. This outperformance is obviously depending on the initial weights and the returns of the stocks in the portfolio. The unequal weights of the B&H portfolio lead to the higher portfolio return in this example.

In scenario 2 the same absolute returns of the stocks in period t+1 are assumed but with reverse algebraic signs. Tables 3 and 4 show the results for the rebalanced and the B&H portfolio.

Table 3: Example: Rebalanced portfolio at the beginning of t+1 – Scenario 2

Stock	**A**	**B**	**C**	**D**	**E**	**Total**
Portfolio weight at t+1	20%	20%	20%	20%	20%	**100%**
Return in period t+1	-25%	10%	20%	-15%	-35%	
Value at t+2	15%	22%	24%	17%	13%	**91%**
Portfolio weight at t+2	16.48%	24.18%	26.37%	18.68%	14.29%	**100%**

Table 4: Example: B&H portfolio at the beginning of t+1– Scenario 2

Stock	**A**	**B**	**C**	**D**	**E**	**Total**
Portfolio weight at t+1	22.94%	16.51%	14.68%	21.10%	24.77%	**100.00%**
Return in period t+1	-25%	10%	20%	-15%	-35%	
Value at t+2	17.20%	18.17%	17.61%	17.94%	16.10%	**87.02%**
Portfolio weight at t+2	19.77%	20.88%	20.24%	20.61%	18.50%	**100.00%**

According to these results, a rebalancing strategy would have led to the following results depending on the different scenarios (see equation (5)):

Table 5: Example: Results of both scenarios

	Scenario 1	Scenario 2
Return Rebalanced ($\bar{r}_{PF}^{g}$)	$\bar{r}_{PF}^{g} = \sqrt{1.09 \times 1.09} - 1 = 9\%$	$\bar{r}_{PF}^{g} = \sqrt{1.09 \times 0.91} - 1 = -0.41\%$
Return B&H ($\bar{r}_{B\&H}^{g}$)	$\bar{r}_{B\&H}^{g} = \sqrt{1.09 \times 1.1298} - 1 = 10.97\%$	$\bar{r}_{B\&H}^{g} = \sqrt{1.09 \times 0.8702} - 1 = -2.61\%$
Rebalancing Return	$RR_H = 9\% - 10.97\% = -1.97\%$	$RR_H = -0.41\% - (-2.61\%) = 2.2\%$

In this simple example the rebalancing return is negative in case of a positive market trend and positive in case of positive market that is followed by a negative market. Considering the B&H portfolio the larger portfolio weight differences (scenario 1) lead to a lower rebalancing return, and vice versa.

Unequal weights in a portfolio mean a larger weighting based concentration compared to an equally weighted portfolio. With a rising price of a single stock its portfolio weight and thus the concentration of the B&H portfolio increases.

Hence, the weight concentration of a B&H portfolio should be relatively high within a portfolio of widely differing stock price movements, and vice versa. It can be determined using the normalized Herfindahl index $H^*(w)$ in the following way (Roncalli, 2014, pp. 126-127):

$$H^*(w) = \frac{n \times H(w) - 1}{n - 1} \tag{17}$$

where $H(w) = \sum_{i=1}^{n} w_i^2$ which is the Herfindahl index associated with w, and n is the number of stocks in the portfolio.

In the case of a portfolio that is regularly adjusted to equal stock weights, the normalized Herfindahl index will be 0:

$$H(w)^{\text{Rebalanced}} = \sum_{i=1}^{n} w_i^2 = n \cdot \left(\frac{1}{n}\right)^2 = \frac{1}{n} \qquad => \qquad H^*(w)^{\text{Rebalanced}} = \frac{n \times \frac{1}{n} - 1}{n - 1} = 0$$

The difference between a rebalanced (equally weighted) portfolio and a B&H portfolio becomes obvious when considering for example two stocks that are developing in the B&H portfolio in such a way that at the end of the period the weights are as follows: $w_1 = 0.8$ and $w_2 = 0.2$. For the rebalanced portfolio applies in this case after rebalancing: $w_1 = w_2 = 0.5$ and hence $H^*(w) = 0$. The B&H portfolio has a $H^*(w)$ of 0.36:

$$H(w)^{B\&H} = \sum_{i=1}^{n} w_i^2 = 0.8^2 + 0.2^2 = 0.68 \qquad => \qquad H^*(w)^{B\&H} = \frac{2 \cdot 0.68 - 1}{2 - 1} = 0.36$$

As a further measure of concentration, the coefficient of variation (CV) can be used. It can be determined in terms of portfolio concentration as follows (Chen and Lai, 2015, p. 271):

$$CV = \frac{\sigma(w_i)}{\mu(w_i)} \tag{18}$$

where $\sigma(w_i)$ is the standard deviation of all stock weights in the portfolio and $\mu(w_i)$ is the mean of all stock weights in the portfolio.

As with the normalized Herfindahl Index, for the coefficient of variation, a higher value means a higher portfolio concentration on a relatively small number of stocks. Mathematically, the coefficient of variation is related to the normalized Herfindahl index as follows (see appendix for the derivation of the formula):

$$H^*(w) = \frac{CV^2}{n-1} \tag{19}$$

A higher portfolio concentration on a few stocks arises when the weights of individual stocks increase due to their relatively good performance, while the weights of the stocks with a relatively low return trend lose weight. Thus, if a portfolio is rebalanced to equal weights after each period, the concentration of this portfolio will be lower than the concentration of a B&H portfolio where the weights are not rebalanced and influence the weights of the following period.

If a B&H portfolio is high (weight) concentrated due to single stocks that performed much better than others over a longer period of time, it can be expected that the portfolio return is higher than the one of the rebalanced portfolio. Hence, the rebalancing return will be lower with a higher concentrated B&H portfolio as shown in the example above. In section 4 it will be tested empirically if this relationship can be generalized.

A certain stock price trend would mean that there is a relatively high autocorrelation between the returns of this stock. According to Chambers and Zdanowicz (2014, pp. 71 and 74), trending (mean-reverting) stock prices should lead to negative (positive) rebalancing returns. Autocorrelation of returns describes the correlation of an asset return with itself over specific time periods ("time lag"). According to Poddig, Dichtl and Petersmeier (2003, p. 99), the empirical autocorrelation c_k at lag k can be expressed in the following way:

$$c_k = \frac{\frac{1}{n-k} \times \sum_{t=k+1}^{n} (r_t - \bar{r}) \times (r_{t-k} - \bar{r})}{\frac{1}{n-1} \times \sum_{t=1}^{n} (r_t - \bar{r})^2} \tag{20}$$

where k is the time lag, n is the number of observations (and at the same time the current point of time or today, respectively), r_t is the return at time t, and $\bar{r}$ is the arithmetic average return.

Using a simple example, Meyer-Bullerdiek (2017, pp. 10-11 and 24-25) showed that a negative (positive) autocorrelation of all assets in a portfolio does not necessarily lead to a positive (negative) rebalancing return. In section 4 it will be tested empirically if there is a certain relationship between the average autocorrelation of the stock returns in a portfolio and the rebalancing return.

4 Empirical Results

In the empirical analysis the Monte Carlo simulation is used to generate weekly logarithmic stock returns. Thus, problems with data specific results can be avoided. It is assumed that these returns are normally distributed. The simulation is based upon a mean weekly logarithmic return of 0.13% and a standard deviation of weekly logarithmic returns of 2.82%. These values are calculated from the data of the German stock index DAX between 29th January 1971 and 27th January 2017. The random numbers are generated with MS Excel.

In each simulation 520 weekly returns are generated for each of the 15 assumed stocks in the portfolio. In total, 1,000 simulations are run so that the study is based on 7.8 million simulated returns.

At the beginning of the analysis (t_0), an equally weighted portfolio worth EUR 1.5 million is assumed, consisting of 15 stocks, each with a market value of EUR 100. Accordingly, the portfolio in t_0 consists of 1,000 shares (or EUR 100,000) of each of the 15 stocks. Two portfolios are considered: a rebalanced portfolio, which will be rebalanced every week to equal weights, and a B&H portfolio, with no adjustments made.

From the simulated (weekly) logarithmic returns, the corresponding prices of the stocks are calculated for 520 periods. Subsequently, for each period the portfolio value is calculated and, for the rebalanced portfolio, the portfolio weights of all stocks are reset to equal weights. For the B&H portfolio, the current weights are recalculated in each period. A detailed example of the way of calculation is provided by Meyer-Bullerdiek (2016, pp. 41-42).

To determine the diversification ratio, the average (weekly) weights are used in the numerator of equation (2) because in this study weekly returns are assumed. The return-to-risk ratio is calculated on the basis of the arithmetic average return of the portfolio (numerator of equation (3)).

Of particular interest is the difference between the diversification ratio (DR) of the rebalanced and that of the B&H portfolio in the respective simulations. For this reason, the average value of this difference over all simulations and the associated standard deviation are calculated. The same applies to the return to risk ratio difference between the values of the rebalanced and the B&H portfolio. For both the diversification ratio difference and the return to risk ratio difference, the significance is determined using a t-test.

Therefore, the following statistic is used (Bleymüller and Weißbach, 2015, p. 135-136, Bruns and Meyer-Bullerdiek, 2013, p. 772):

$$t = \frac{\overline{DRD} - \mu}{\sigma_{DRD}} \times \sqrt{n} \tag{21}$$

where $\overline{DRD}$ is the average diversification ratio difference, σ_{DRD} is the standard deviation of the diversification ratio difference, μ is the specified value (here it is taken to be 0), and n is the sample size (which is 1,000 in this study).

This statistic can be used in the presence of a normally distributed population and unknown variance of the population. The null hypothesis and the alternative hypothesis are defined as follows:

Null hypothesis: $\mu = 0$
Alternative hypothesis: $\mu > 0$

Accordingly, it is tested whether the diversification ratio difference is significantly positive, based on a significance level (error rate) of $\alpha = 5\%$ which is often used in the economic and social sciences. Correspondingly, the relevant critical value for t can be taken from the t-distribution table. At values below this critical value, the null hypothesis is maintained; because then it cannot (significantly) be rejected. If the values are above the critical value, it can be assumed that the diversification ratio difference is significantly positive (Poddig, Dichtl and Petersmeier, 2003, pp. 338-339, 344, and 767).

According to the relationship between rebalancing and the utility value for a certain investor, different degrees of risk (equation 4) are used: A=2, 4, 6, 8, and 10.

Regarding the calculation of the concentration of the B&H portfolio, the normalized Herfindahl Index and the coefficient of variation are used. These values are calculated using the final weights (at period 520) as well as the average weights of the individual stocks in the B&H portfolio over all 520 periods.

The rebalancing return is calculated according to equation (5). To determine to what extent the rebalancing return is significantly positive a t-test is used.

To analyze the relationship between the rebalancing return and the autocorrelation of returns in the respective portfolios, the average autocorrelations ($\bar{c}_k$) of the stocks in the portfolio are used (Munkelt, 2008, p. 100):

$$\bar{c}_k = \frac{1}{n} \cdot \sum_{i=1}^{n} c_{k_i} \tag{22}$$

where n is the number of stocks in the portfolio and c_k is the autocorrelation at lag k. It is examined if there is a correlation between the average autocorrelation of the stocks of a portfolio and the rebalancing return of this portfolio for different lags (k). For negative autocorrelations of stock returns, the rebalancing return should be positive, i.e. rebalancing should pay off, and vice versa (Hayley et al., 2015, p. 14.).

A correlation between the autocorrelations of the returns and the weights of the respective stocks in the portfolio cannot be determined because a positive autocorrelation can lead to both increasing and decreasing weights.

Tables 6a and 6b present for all 1,000 simulations the averages and the associated standard deviations, indicated in brackets.

Table 6a: Results of the Monte-Carlo-Simulation

	Weekly rebalancing	**B&H**
Average diversification ratio of portfolio and corresponding standard deviation	3.8727 (11.5076%)	3.5359 (16.0031%)
Average return-to-risk ratio and corresponding standard deviation	23.3740% (4.3006%)	21.2906% (4.1834%)
Average utility value for A=2 and corresponding standard deviation	0.1649% (0.0309%)	0.1640% (0.0348%)
Average utility value for A=4 and corresponding standard deviation	0.1596% (0.0309%)	0.1576% (0.0346%)

Table 6b: Results of the Monte-Carlo-Simulation

	Weekly rebalancing	**B&H**
Average utility value for A=6 and corresponding standard deviation	0.1543% (0.0309%)	0.1512% (0.0344%)
Average utility value for A=8 and corresponding standard deviation	0.1490% (0.0309%)	0.1448% (0.0343%)
Average utility value for A=10 and corresponding standard deviation	0.1437% (0.0309%)	0.1383% (0.0342%)
Average rebalancing return and corresponding standard deviation	0.0003988% (0.014945%)	0 (0)
Average arithmetic mean return of portfolio and corresponding standard deviation	0.1703% (0.0309%)	0.1704% (0.0350%)
Average geometric mean return of portfolio and corresponding standard deviation	0.1676% (0.0309%)	0.1672% (0.0349%)
Average standard deviation of portfolio returns and corresponding standard deviation	0.7291% (0.0224%)	0.8000% (0.0386%)
Average normalized Herfindahl index (based upon the weights at the end of the 520 periods) and corresponding standard deviation	0 (0)	3.0420% (1.8863%)
Average coefficient of variation (based upon the weights at the end of the 520 periods) and corresponding standard deviation	0 (0)	62.9612% (17.1650%)
Average normalized Herfindahl index (based upon the average weights over the 520 periods) and corresponding standard deviation	0 (0)	0.9455% (0.4655%)
Average coefficient of variation (based upon the average weights over the 520 periods) and corresponding standard deviation	0 (0)	35.4304% (8.2690%)

At first, the difference between the diversification ratio of the rebalanced portfolio (DR^{reb}) and that of the B&H portfolio ($DR^{B\&H}$) is considered. In both cases, the average diversification ratio is greater than 3 with a higher value for the rebalanced portfolio. The average difference is 0.3368 (with a standard deviation of 14.0137%).

The extent to which the average diversification ratio difference is significantly positive can be tested with a t-test. In this case, t has the following value:

$$t = \frac{\overline{DRD} - \mu}{\sigma_{DRD}} \times \sqrt{n} = \frac{0.336779 - 0}{0.140137} \times \sqrt{1{,}000} = 75.9963$$

According to the t-distribution table, the critical value is 1.646 in this case (Poddig, Dichtl and Petersmeier, 2003, p. 767). Since the empirically determined t-value is much greater than the critical value, the null hypothesis can be rejected. Thus, the average diversification ratio difference is significantly positive. Accordingly, in this study the diversification ratio of the rebalanced portfolio is significantly larger than that of the B&H portfolio.

Figure 1 shows the frequency distribution of the diversification ratio difference ($DR^{reb} – DR^{B\&H}$).

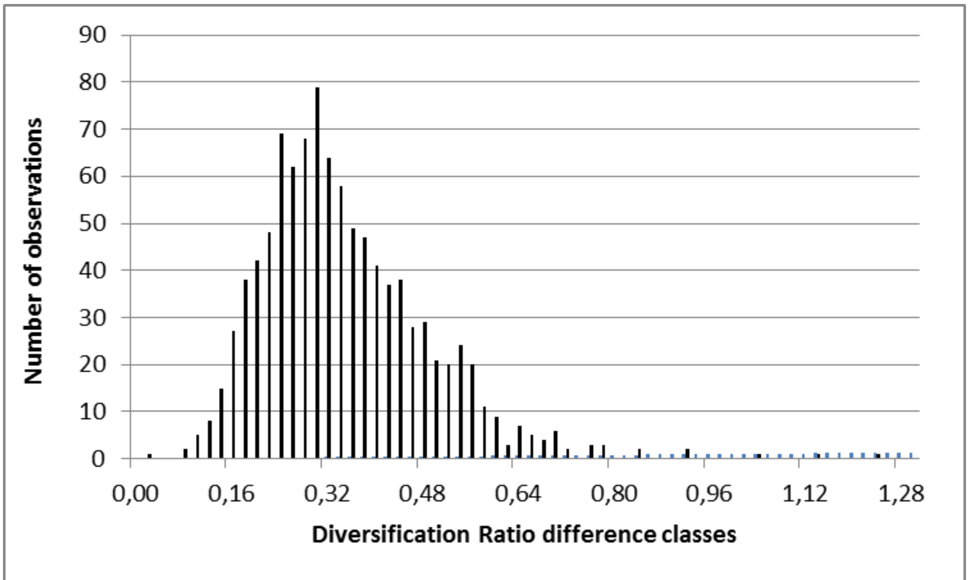

Figure 1:Distribution of the diversification ratio difference (DR^{reb} - $DR^{B\&H}$) for a portfolio of 15 stocks with normally distributed returns. 1,000 simulations over 520 rebalancing periods are considered.

Furthermore, the comparison of the return to risk ratio of the rebalanced portfolio (rtr^{reb}) and the B&H portfolio ($rtr^{B\&H}$) also leads to significant values. The difference between these values averages 2.0834% over all 1,000 simulation runs (with a standard deviation of 1.4266%). With a t-value of 46.18 and a significance level (error rate) of 5%, it is also significantly positive. Thus, the return to risk ratio of the rebalanced portfolio is significantly greater than that of the B&H portfolio. This is also reflected in Figure 2.

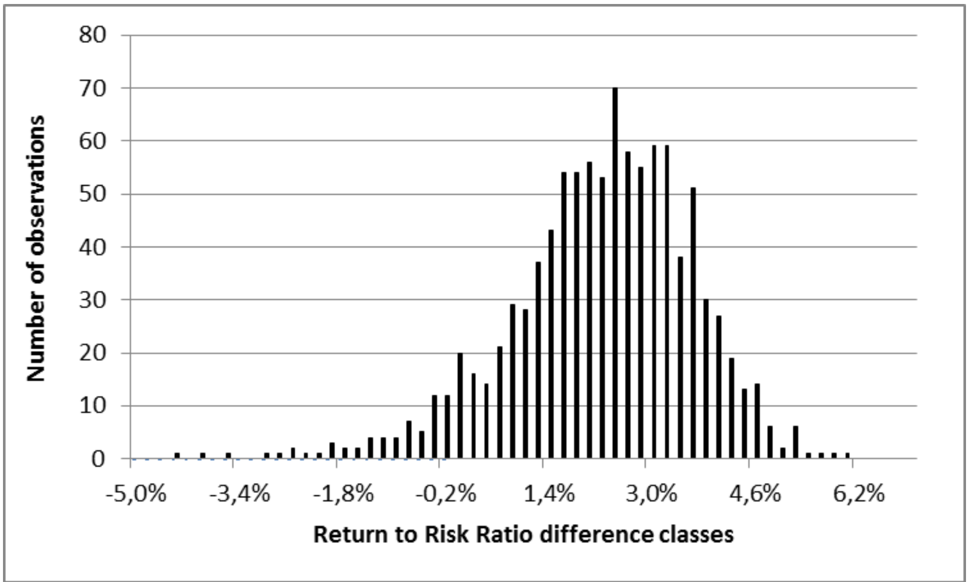

Figure 2:Distribution of the return to risk ratio difference (rtr^{reb} - $rtr^{B\&H}$) for a portfolio of 15 stocks with normally distributed returns.
1,000 simulations over 520 rebalancing periods are considered.

Now, the difference between the utility value of the rebalanced portfolio (U^{reb}) and that of the B&H portfolio ($U^{B\&H}$) is considered for different levels of risk aversion. The results are presented in Table 7.

Table 7: Monte-Carlo-Simulation: Utility value differences, the corresponding standard deviations and t-values.

Risk Aversion degree	**U^{reb} - $U^{B\&H}$ (average)**	**Standard deviation**	**t-value**
A = 2	0.0009%	0.0148%	2.0293
A = 4	0.0020%	0.0144%	4.4873
A = 6	0.0031%	0.0140%	7.0706
A = 8	0.0042%	0.0137%	9.7820
A = 10	0.0053%	0.0133%	12.6232

For all used degrees of risk aversion the average utility value of the rebalanced portfolio is significantly greater than that of the B&H portfolio. The difference is the greater the higher the risk aversion.

The rebalanced portfolio has a rebalancing return of 0.0003988% on average over all 1,000 simulation runs (with a standard deviation of 0.0149450%). Figure 3 shows the frequency distribution of the rebalancing return.

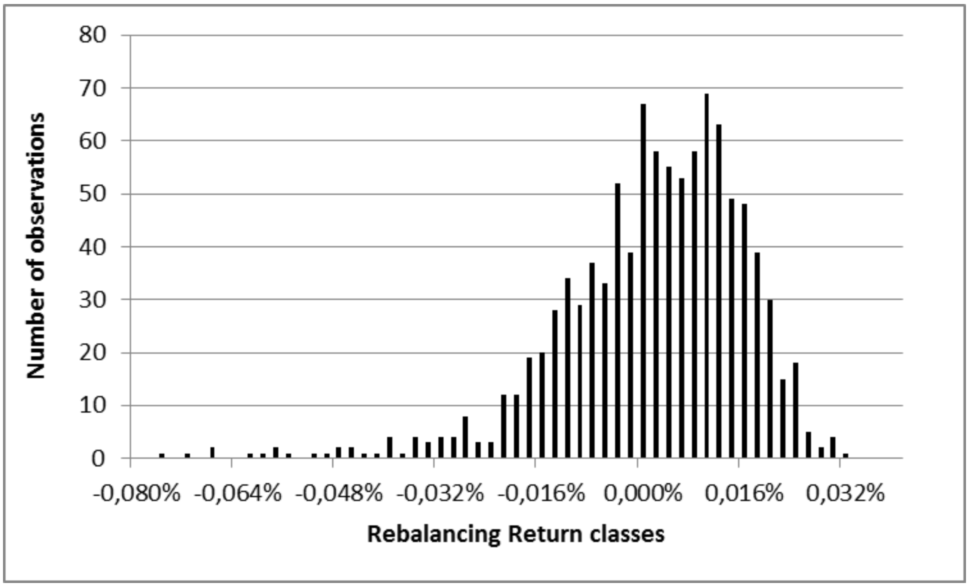

Figure 3:Distribution of the rebalancing return for a portfolio of 15 stocks with normally distributed returns. 1,000 simulations over 520 rebalancing periods are considered.

This histogram is comparable to the findings of Dubikowskyy and Susinno (2015, p. 232) for a portfolio of two uncorrelated assets with normally distributed returns (with μ = 0 and σ = 2% per period). They considered 100,000 simulations over 10 rebalancing periods and found a strong negative skew with frequent small positive rebalancing returns which will be offset by rare but large negative returns.

The extent to which the average rebalancing return is significantly positive can be tested with a t-test. In this case, t has a value of 0.8439 which is lower than the critical value of 1.646. Hence, the null hypothesis cannot be rejected. The (positive) average rebalancing return is therefore not significantly positive. Accordingly, regular rebalancing in this study does not lead to significantly better geometric returns than the B&H strategy.

The values for the normalized Herfindahl Index (Table 6b) show that the B&H portfolio has a clearly positive (weight) concentration. The minimum value based on the weights at the end of the 520 periods is 0.4913% for 1,000 simulation runs. If the average weights over the 520 periods are used, the minimum value is 0.1592%.

A comparison between the rebalancing return and the weight concentration of the B&H portfolio is shown in Table 8.

Table 8: Monte-Carlo-Simulation: correlation between the rebalancing return and the weight concentration of the B&H portfolio

	Correlation
Correlation between rebalancing return and normalized Herfindahl index (based upon the weights at the end of the 520 periods)	-0.876938
Correlation between rebalancing return and the coefficient of variation (based upon the weights at the end of the 520 periods)	-0.910835
Correlation between rebalancing return and normalized Herfindahl index (based upon the average weights over the 520 periods)	-0.739145
Correlation between rebalancing return and the coefficient of variation (based upon the average weights over the 520 periods)	-0.737899

The values show a significant negative correlation between the rebalancing return and the weight concentration of the B&H portfolio. The higher this portfolio concentration, the lower the rebalancing return. Hence, when there are severe weight differences in the B&H portfolio compared to the rebalanced portfolio, regular rebalancing seems not to be beneficial.

The analysis of the relationship between the rebalancing return and the average autocorrelation of stock returns within a portfolio ($\bar{c}_k$) does not produce clear results, as shown in Table 9.

The theoretically expected negative correlation between the rebalancing return and the autocorrelation does not occur in this study at every lag. In addition, the (absolute) correlation values are very low. These results may be due to the use of average autocorrelations per portfolio. On the other hand, it has to be considered that independent returns (that should not be autocorrelated) are used in this study.

Table 9: Monte-Carlo-Simulation: correlation between rebalancing return and the average autocorrelation of stock returns in the portfolio

	Correlation
Correlation between rebalancing return and average autocorrelation of stock returns at lag 1	-0.026887
Correlation between rebalancing return and average autocorrelation of stock returns at lag 2	0.012473
Correlation between rebalancing return and average autocorrelation of stock returns at lag 3	-0.033324
Correlation between rebalancing return and average autocorrelation of stock returns at lag 4	0.002331

5 Conclusion

In this study, it is firstly discussed how rebalancing affects portfolio diversification, risk-adjusted return and the utility value for a certain investor – each compared to the B&H portfolio. To measure the portfolio diversification the diversification ratio is used. In addition to this, the return to risk ratio and a popular function for calculating the utility score are used whereas it is assumed that investors can assign a utility score to different portfolios that are based upon risk and return. Secondly, the relationship between the weight-based concentration of the B&H portfolio and the success of a rebalancing strategy is explored. For this purpose, it is shown how the portfolio weight of a special stock is depending on the initial weights of all stocks at the beginning of the holding period and on the returns of all stocks. In case of an initially equally weighted portfolio the weight at the end of the period is not depending on the initial weight at the beginning of the period, but on the return of the stock, the average return of the other stocks, and on the number of stocks in the portfolio. If the portfolio is not equally weighted at the beginning of the period (like a B&H portfolio) the weights of all stocks have to be considered. Thus, a B&H portfolio has a larger weight concentration which in this study is determined by the normalized Herfindahl index and the coefficient of variation.

In the empirical analysis the Monte Carlo simulation is used to generate weekly logarithmic stock returns that are normally distributed. In each of the 1,000 simulations 520 weekly returns are generated for each of the 15 assumed stocks in the portfolio. It is supposed that the 15 stocks are initially equally weighted for both, the rebalanced portfolio and the B&H portfolio.

The empirical results show that the diversification ratio of the rebalanced portfolio turns out to be significantly greater than that of the B&H portfolio. According to this measure, a rebalanced portfolio is better diversified. The comparison of the return to risk ratio of the rebalanced portfolio and the B&H portfolio also leads to significant differences. The return to risk ratio of the rebalanced portfolio is significantly greater than that of the B&H portfolio. These findings are supported by the utility value difference between the rebalanced and the B&H portfolio for different levels of risk aversion. For all degrees of risk aversion used in this study the average utility value of the rebalanced portfolio is significantly greater than that of the B&H portfolio. This difference is the greater the higher the risk aversion.

Besides, the rebalanced portfolio has a slightly positive rebalancing return, but it is not significant at a significance level (error rate) of 5%. The analysis of the relationship between the rebalancing return and the average autocorrelation of stock returns within a portfolio does not produce clear results.

Furthermore, the B&H portfolio has a positive (weight) concentration which is much greater than zero which is the concentration of the rebalanced portfolio in this study because of equal weights at the beginning of every period. There is a strong negative correlation between the rebalancing return and the weight concentration of the B&H portfolio. The higher this portfolio concentration, the lower the rebalancing return. Hence, when weight differences in the B&H portfolio are growing, regular rebalancing of the equally weighted portfolio seems not to be beneficial.

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Appendix

Derivation of equation (19)

$$CV^2 = \frac{\sigma^2(w_i)}{\mu^2(w_i)} = \frac{\frac{1}{n}\cdot\sum_{i=1}^{n}(w_i-\mu)^2}{\mu^2} = \frac{\frac{1}{n}\cdot\left(\sum_{i=1}^{n}w_i^2 - n\cdot\mu^2\right)}{\mu^2} = \frac{\frac{1}{n}\cdot\sum_{i=1}^{n}w_i^2 - \mu^2}{\mu^2} = \frac{\frac{1}{n}\cdot\sum_{i=1}^{n}w_i^2}{\mu^2} - 1$$

$$\Leftrightarrow \quad CV^2 = \frac{\frac{1}{n}\cdot\sum_{i=1}^{n}w_i^2}{\left(\frac{1}{n}\cdot\sum_{i=1}^{n}w_i\right)^2} - 1 = \frac{\frac{1}{n}\cdot\sum_{i=1}^{n}w_i^2}{\frac{1}{n^2}\cdot 1} - 1 = n\cdot\sum_{i=1}^{n}w_i^2 - 1 = n\cdot H(w) - 1$$

$$=> \quad H^*(w) = \frac{n\cdot H(w) - 1}{n-1} = \frac{CV^2}{n-1}$$