

Determination of Incompressibility, Elasticity and the Rigidity of Surface Soils and Shallow Sediments from Seismic Wave Velocities

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Abstract

Geophysical investigation was performed on the topsoil of the Benin Formation in Eket Local Government Area in order to access the degree of top soil stability based on elastic constants (Young modulus, Shear modulus and Bulk modulus). The seismic refraction method employing P-wave and S-wave sources were used to generate seismic energies that propagated through the subsurface. A 12-channel seismograph with signal stacking ability was used together with high frequency (100 Hz) geophones on the top-soil. The geophone intervals were set to 5 m at all the locations. The results of the findings based on these parameters were Young's Modulus E ($-40.772 \times 10^8 \text{ N/m}^2$ to $16.1481 \times 10^8 \text{ N/m}^2$); Bulk Modulus k ($-0.7964 \times 10^8 \text{ N/m}^2$ to $7.6896 \times 10^8 \text{ N/m}^2$) and Shear Modulus μ ($1.3751 \times 10^8 \text{ N/m}^2$ to $7.0209 \times 10^8 \text{ N/m}^2$). A reasonable thickness of the top layer is porous, swampy, air-filled and weak. Excavation and refilling of the top and weathered zone with resilient geomaterials should be effective, and vibration test be carried out to ascertain the degree of reinforcement.

Keywords: Seismic refraction, Young modulus, Shear modulus, Bulk modulus, Eket,

1 Introduction

Application of seismic refraction method ranges from engineering site investigation surveys to large scale experiments designed to study the structure of the earth, ground search, mineral exploration and in petroleum exploration. Seismic refraction is employed in the determination of the dynamics elastic modulus of the subsurface layer to obtain information on the elastic properties of that layer.

Seismic refraction method utilizes seismic energy that returns to the surface after traveling through the ground along refracted ray path. This method is normally used to locate refracting interfaces separating layers of the different seismic velocities [1].

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Interpretation of these measurement and data can reveal how the physical properties of the earth vary vertically and laterally. In this work, seismic refraction is used to determine the elastic moduli (Young modulus, Bulk modulus, Shear modulus and Lamé's constant) and as well infer the degree of stability of engineering foundation based on these elastic parameters.

2 Theoretical Framework

The velocity of propagation of the body wave in any material is given by

$$V = \sqrt{\frac{\text{approximate elastic moduli of the material}}{\text{density of the material}}} \quad (1)$$

The velocity V_p of a compressional body waves which involves a uniaxial compressional strain is given by

$$V_p = \left(\frac{\varphi}{\rho}\right)^{\frac{1}{2}} \quad (2)$$

$$\varphi = K + \frac{4}{3}\mu \quad (3)$$

therefore,

$$V_p = \sqrt{\frac{K + \frac{3}{4}\mu}{\rho}} \quad (4)$$

Where K = Bulk modulus, μ = Rigidity Modulus, ρ = density of the medium. The velocity of the shear body wave V_s which involve a pure strain is given as

$$V_s = \sqrt{\frac{\mu}{\rho}} \quad (5)$$

From (5), compressional waves travel faster than shear wave in the same medium. The ratio V_p/V_s in any medium is determined by the value of Poisson's ratio for the material [2]. Body waves are non-dispersive, that is all frequency components in a wave train travel through any material at the same velocity determined only by the elastic moduli and density of the material [3].

The ratio V_p/V_s known as the lithology discriminator aids in the determination of elastic properties. In considering the homogeneous and isotropic solid material, only three elastic constants are required to describe the mechanical properties of the system. These are bulk modulus, Young modulus and Shear modulus. The estimation of these parameters are done through the use of universal relations among K, σ, μ, E and $\frac{V_p}{V_s}$ ratio.

$$K = \frac{2\mu(1+\sigma)}{3(1-2\sigma)} \quad (6)$$

$$E = 2\mu [1 + \sigma] \quad (7)$$

$$\mu = \frac{E}{2(1 + \sigma)} \quad (8)$$

and

$$\frac{\left[\left(\frac{V_p}{V_s} \right)^2 - 2 \right]}{\left[2 \left(\frac{V_p}{V_s} \right)^2 - 2 \right]} \quad (9)$$

Generally, there is correlation between seismic velocity, density and ripability of subsurface materials.

3 Location and Geology of the Study Area

The study area, Eket lies between latitudes 4°37' and 4°7'N and longitudes 7°38' and 8°00'E. It has an estimated area of 214 km² (Figure 1). The study area belongs to the low-lying coastal deltaic plains of southern Nigeria [4]. The terrain is virtually flat to gently undulating, sloping generally towards the Atlantic Ocean. Elevation varies from about 100 to 120 m at the northern part of the study area to near sea level at the southern part [5]. The surface drainage basin within the study area is mainly due to the Qua Iboe River which drains the western part [6]. Geologically, the study area falls within the Niger Delta area.

The geologic formation in the Niger Delta area is made up of the Akata Formation (shales, intercalated sands and sandstones), the Agbada Formation (sands and sandstones intercalated with shales) in the middle and the Benin Formation (Coarse grained, gravelly sands with minor intercalations with clay) at the top [7]. However, only the Benin Formation otherwise called the coastal plain sands is exposed in the study area where the investigation concentrates. The coastal plain sand covers 80 percent of the area and forms the major aquiferous and foundation zones of the study area. The area is generally porous and permeable and this is usually interrupted by clay-sand sequence at different depths [8] [9].

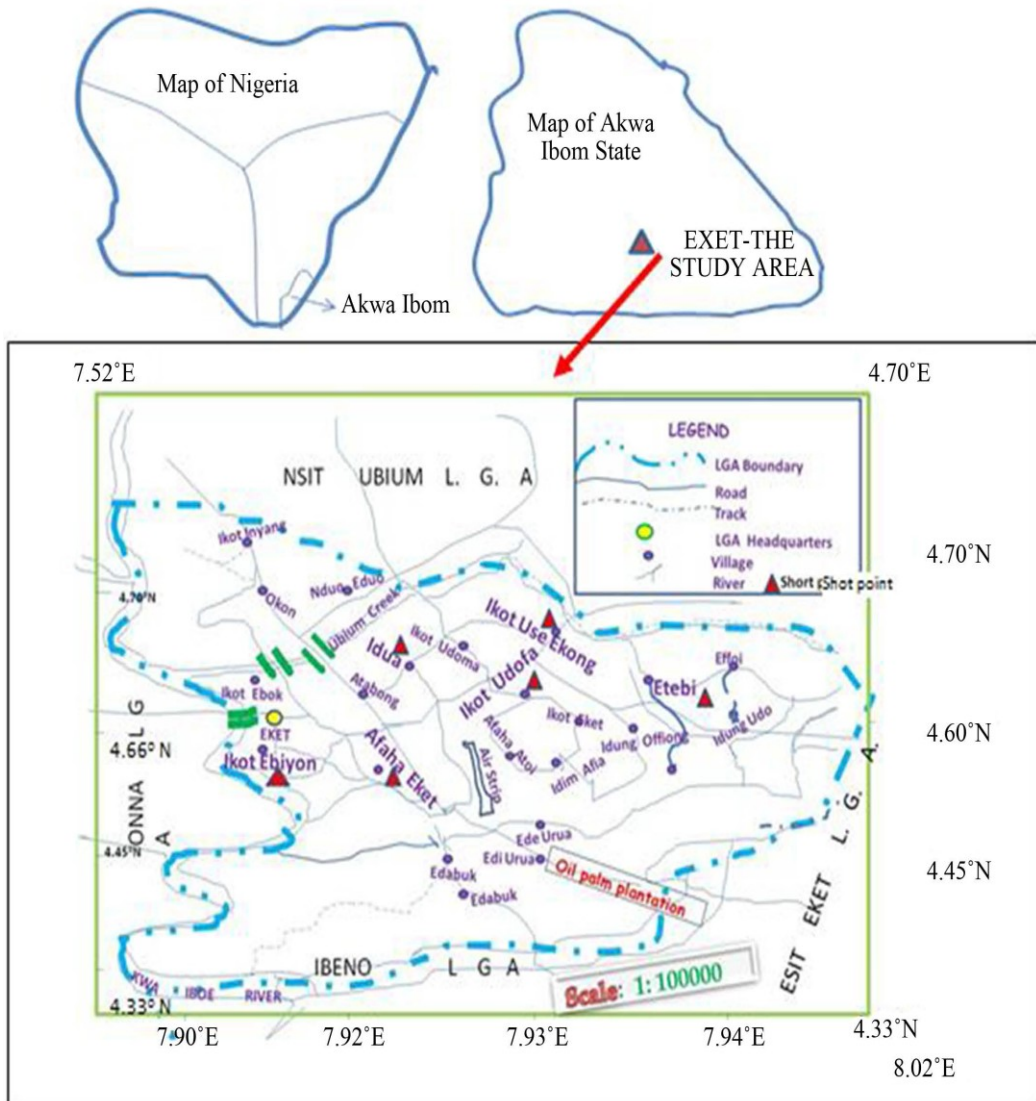


Figure 1: Location map of study area

4 Methodology

The seismic study was based on recording the travel time of an elastic wave travelled through the ground, refracted from a subsurface, and received via geophones on the surface. Seismic P- and S-waves were recorded using vertical and horizontal components geophones (100 Hz), laid along a line with 5 m spacing, and connected to a 12-channel digital recorder. The required energy for P-wave measurement was created by a sledge hammer hitting the wooden cone at the shot points. The S-wave was created by hitting the ends of a flat-lying wooden timber loaded by a person, the weight of which increases the friction and the contact area of the timber with the ground surface [10].

The data obtained from the seismograph (seismogram) were processed using Pickwin software to obtain the arrival time for P-wave and S-wave. With the geophone separations of 5 m interval, T-X graph were plotted for the different locations using IX Refrax software and the inverse of the slope were obtained as velocity for each of the layer penetrated. The Pick-win software helped in picking the arrival times while the IX Refrax directly converted the slope into Velocity for the different layers. The IX Refrax also gave the depth of each of the layers penetrated.

5 Results and Discussion

The processing of the velocity gave the mean p-wave and s-wave velocity as shown in Table 1. Using the mean layer velocities, the V_p/V_s ratios were estimated (table 1). Young modulus, Shear modulus and Bulk modulus were computed (Table 2). As indicated in Table 2, the ratio of elasticity (E) to rigidity (μ) and the ratio of incompressibility (K) to rigidity (μ) were calculated in order to decipher the elastic properties of the topsoil that carries the engineering load.

Table 1: Summary of P-wave, S-wave and V_p/V_s ratio

Location/ name	Lat ⁰	Long ⁰	Layer	V _p mean	V _s Mean	V _p /V _s
Idua	4.6760	7.9256	L ₁	357.5	303.0	1.1709
			L ₂	563.5	325.0	1.5492
AfahaEket	4.6800	7.9145	L ₁	350.0	333.5	1.0495
			L ₂	650.0	410.5	1.5834
IkotEbiyon	4.6667	7.9147	L ₁	369.5	338.0	1.0932
			L ₂	462.0	439.5	1.0512
Ikot Use Ekong	4.6636	7.9156	L ₁	383.0	270.0	1.4185
			L ₂	509.0	430.5	1.1823
IkotUdofia	4.6206	7.9325	L ₁	373.5	363.0	1.0289
			L ₂	861.0	552.5	1.5584
Etebi	4.6125	7.9419	L ₁	297.0	250.0	1.1880
			L ₂	541.0	389.5	1.3890

Table 2: Summary of elastic properties in the study area

Location/ name	Lat ⁰	Long ⁰	Layer	μ (N/m ²)	E (N/m ²)	K(N/m ²)	E/ μ	K/ μ
Idua	4.6760	7.9256	L ₁	2.0198	0.9093	0.1189	0.4137	0.5887
			L ₂	2.9294	5.5531	2.5918	2.2858	1.0668
AfahaEket	4.6800	7.9145	L ₁	2.4469	-16.7891	-0.5680	-6.8613	-0.2321
			L ₂	3.8757	9.0560	4.5503	2.3366	1.1741
IkotEbiyon	4.6667	7.9147	L ₁	2.5134	-5.3435	-0.3475	-2.1260	-0.1383
			L ₂	4.4427	-28.9842	1.0144	-6.5240	0.2283
Ikot Use Ekong	4.6636	7.9156	L ₁	1.6038	3.2397	1.1019	2.0200	0.6871
			L ₂	4.2626	2.0742	0.2751	0.4866	0.0645
IkotUdofia	4.6206	7.9325	L ₁	2.8989	-40.7724	-0.7964	-14.0648	-0.2747
			L ₂	7.0209	16.1481	7.6896	2.3000	1.0952
Etebi	4.6125	7.9419	L ₁	1.3751	3.3167	1.8802	2.4120	1.3673
			L ₂	3.4893	6.7134	1.9002	1.9240	0.5446

The parameters were contoured to display the distribution of the geomaterials of the top soil at the indicated depths (Figures 2 to 9). The lithology discriminator (V_p/V_s ratio) in the first layer is low in parts of the North and South. However, higher values are seen at the North West of the study area (Figure 2). In the second layer, higher values of V_p/V_s ratio is obtained in the Northeastern and in the Southeastern region of the study area (Figure 3). Generally, the V_p and V_s were found to be close in most positions and this makes majority of the V_p/V_s ratios to be less than $\sqrt{2}$. The values of V_p/V_s ratios really affect the elastic parameters and in some cases resulted in negative values.

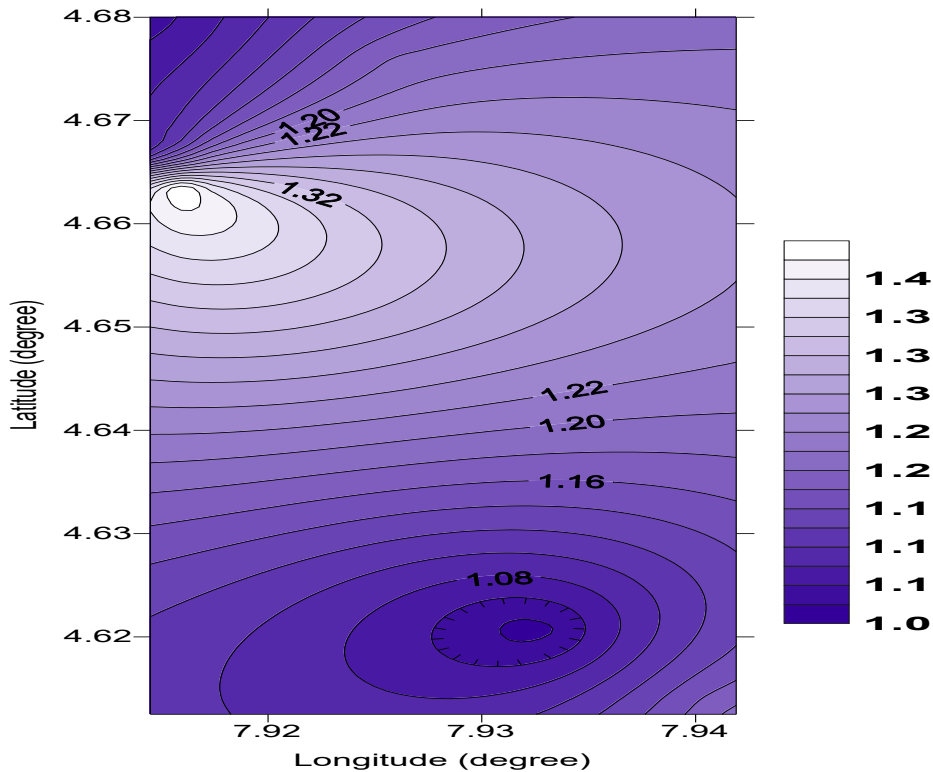


Figure 2: 2-D contour map of layerone V_p/V_s ratio in the study area

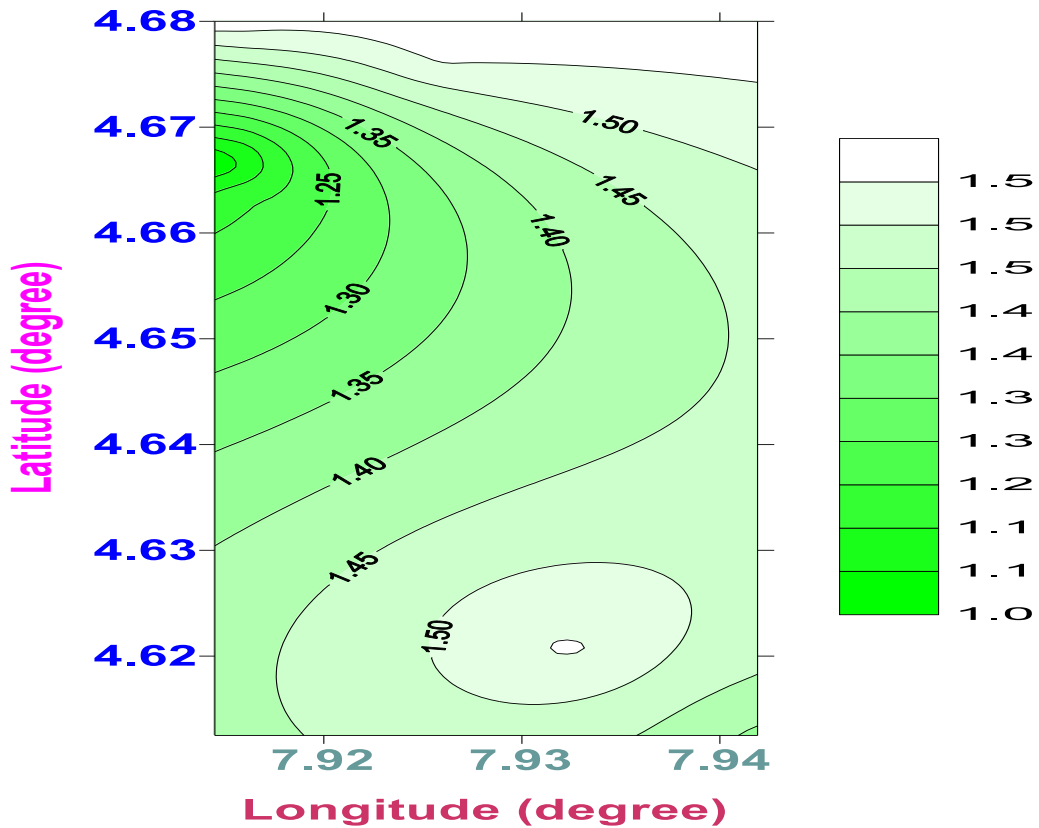


Figure 3: 2-D contour map of layer two V_p/V_s ratio in the study area

For Shear modulus, 3-D plots have been used to show the distribution in the first and second layers (Figures 4 and 5). For layer one, the shear modulus is higher in the Southern region where as in the North and Northeastern region, lower values are obtained. In layer two, the same pattern of increment is also observed for shear modulus.

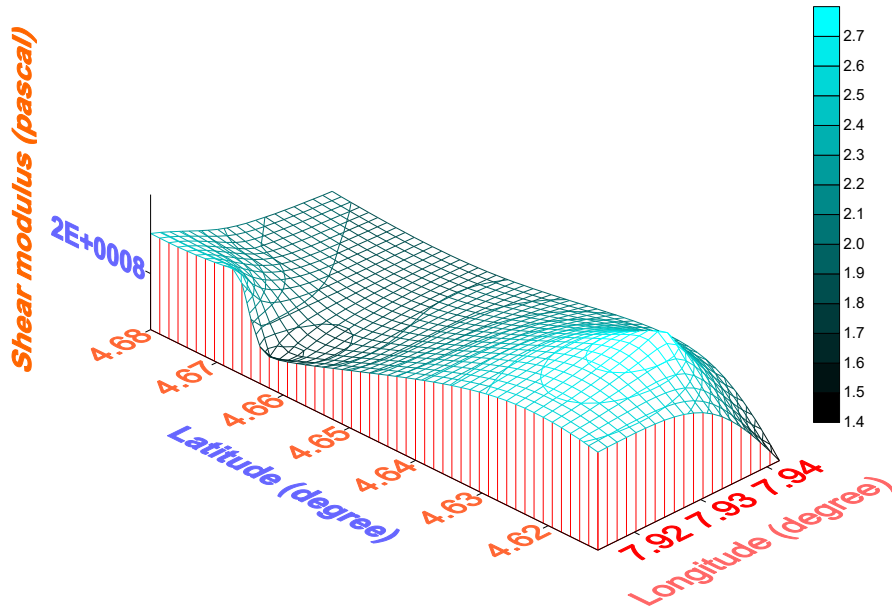


Figure 4: 3-D contour map of layer one Shear modulus in the study area

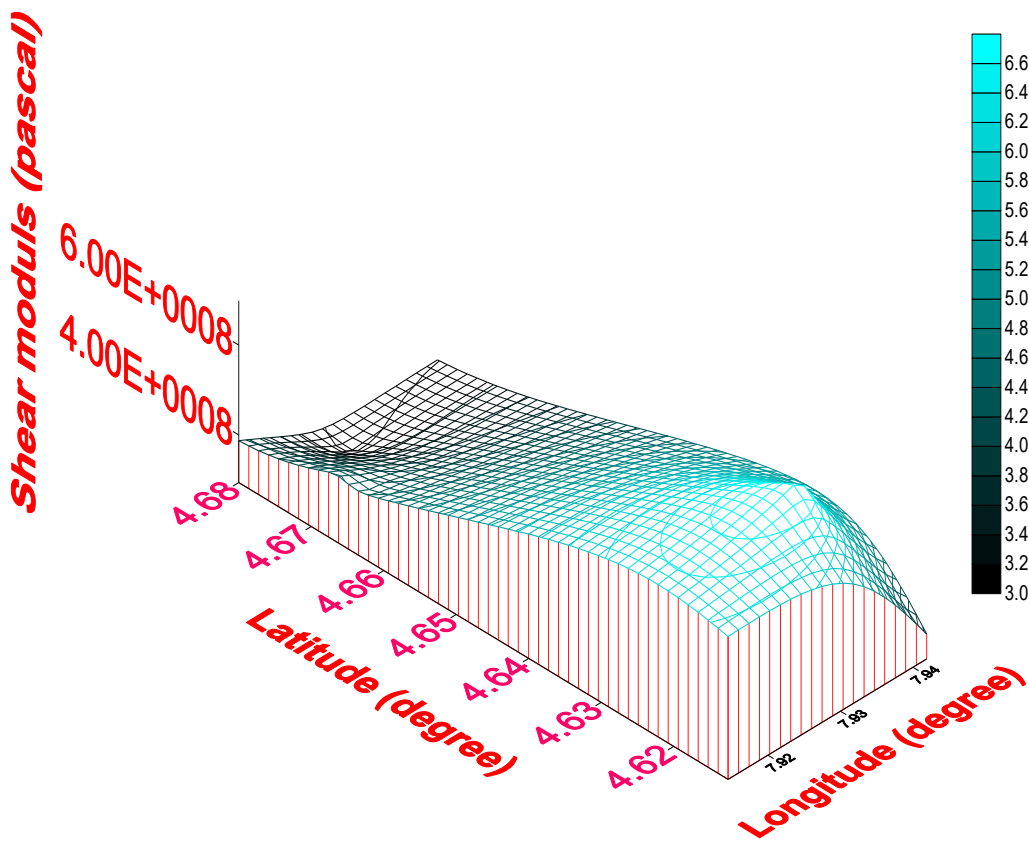


Figure5: 3-D contour map of layer two Shear modulus in the study area

Young's modulus in layer one is higher in the Northern zone where as in the Southern zone kit is lower (Figure 6). The lower value in the Southern region is attributed to the nature of the lithology which is observed to be swampy and water filled. In the second layer, the Young's modulus seems to be uniform except in the North Western zone where a lower value is observed in the 3-D plot (Figure 7).

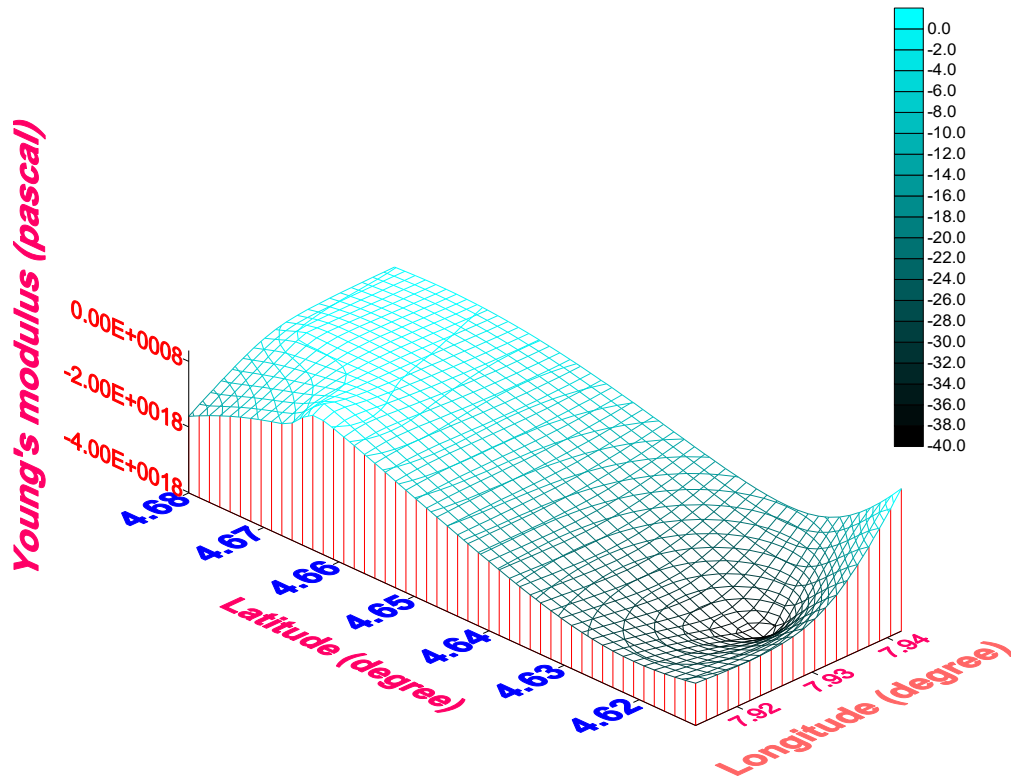


Figure 6: 3-D contour map of layer one Young's modulus in the study area

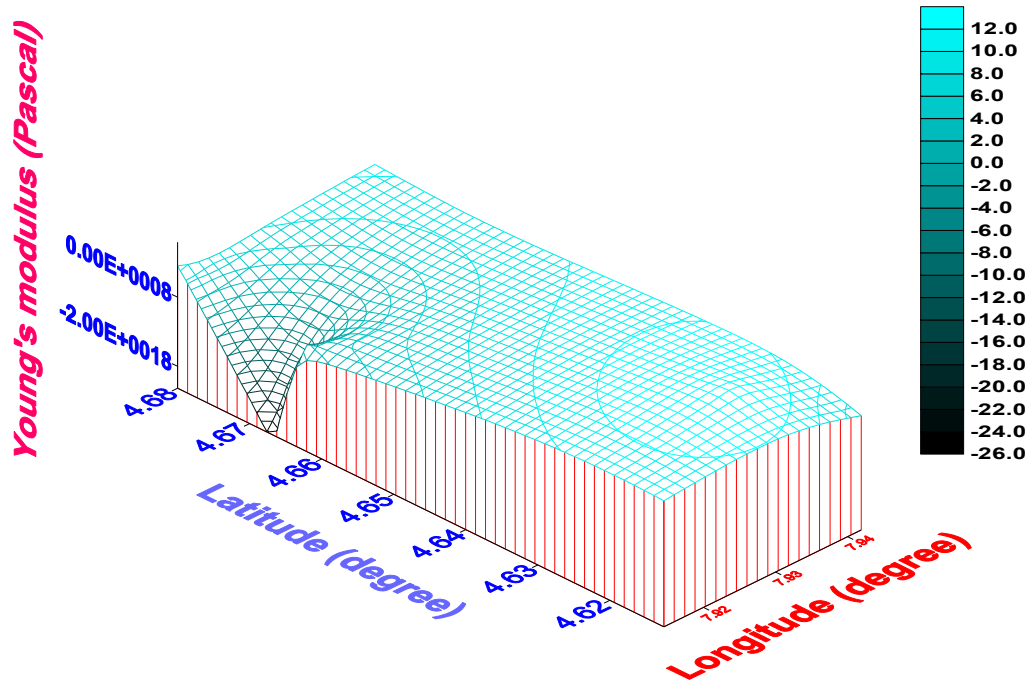


Figure 7: 3-D contour map of layer two Young's modulus in the study area

Bulk modulus according to Figure 8 is low in the first layer. However in the second layer (Figure 9), the bulk modulus seems to be higher in the Southern zone. The ratio distribution equally shows the ratio of elasticity to rigidity.

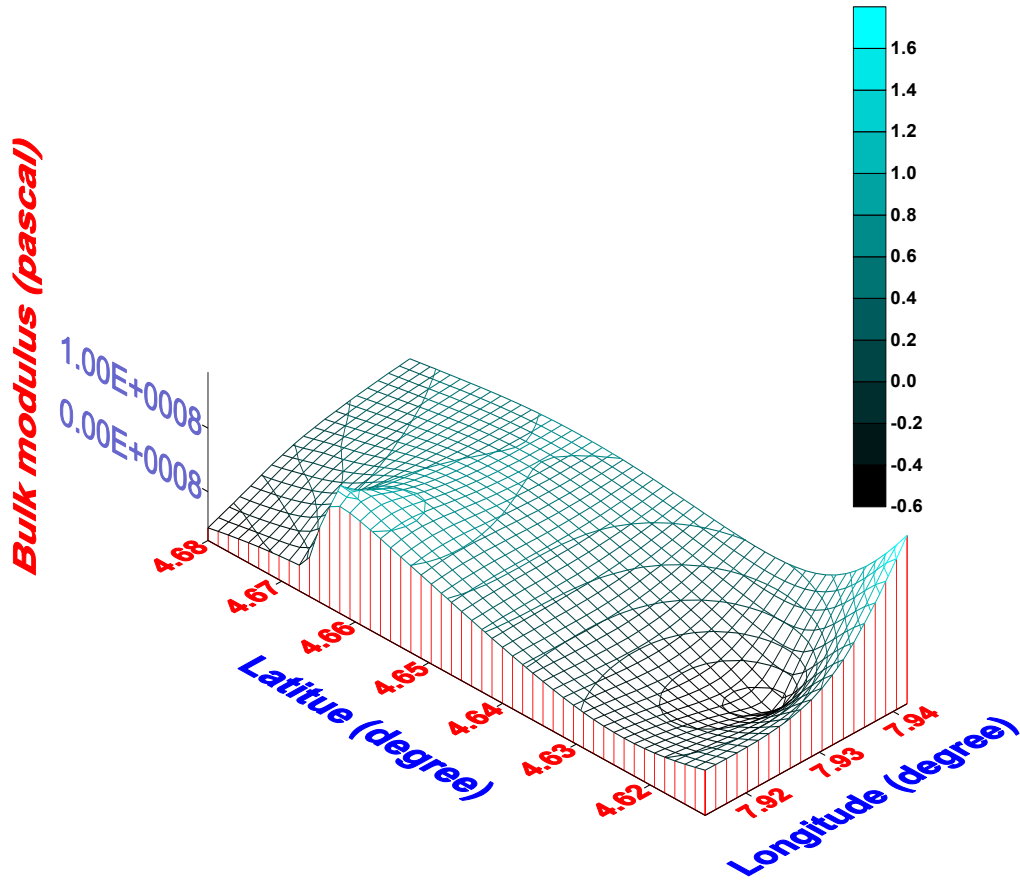


Figure 8: 3-D contour map of layer one Bulk modulus in the study area

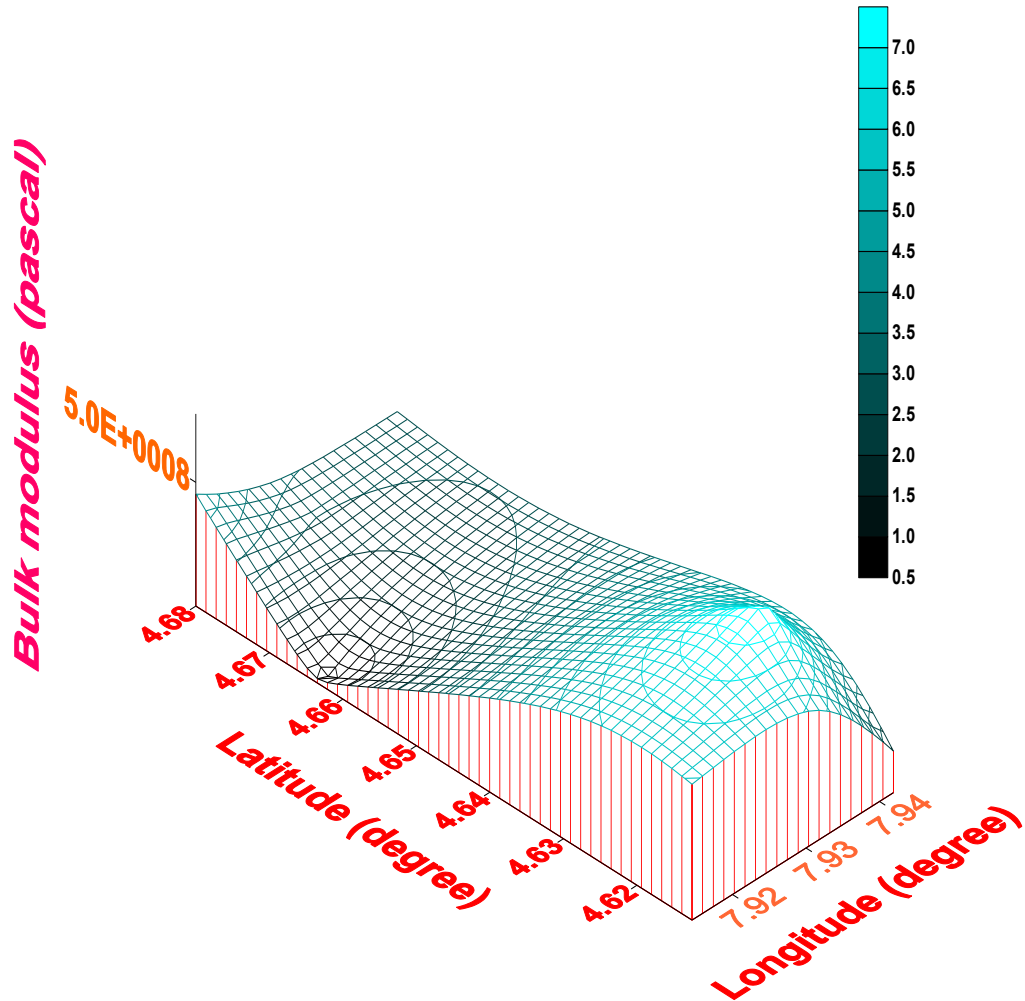


Figure 9: 3-D contour map of layer two Bulk modulus in the study area

6 Conclusion

Geophysical investigation was performed on the topsoil of the Benin Formation in Eket Local Government Area in order to access the degree of top soil stability based on the elastic constants (Young modulus, Shear modulus and Bulk modulus). The results obtained show that reasonable thickness of the top layer is porous, swampy, air-filled and weak according to the determined velocities. Due to the possibility of lithologic changes, wildcat engineering use of the top and weathered soil for construction (road and building) should be discouraged. Rather, geophysical and geological information should be the guideline for engineering construction in the study area. Excavation and refilling of the top and weathered zone with resilient geomaterials should be effective, and vibration test be carried out to ascertain the degree of reinforcement.

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