A Realistic Theory of Soils Consolidation

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Abstract

The consolidation behavior of soils is usually predicted by making use of the convectional theory of consolidation proposed by Terzaghi.Laboratory observations of the consolidation behavior exhibit discrepancies between the theory and the results. These discrepancies are usually attributed to the secondary effects that occur during primary consolidation. On the other hand, Terzaghi's theory presupposes the constancy of permeability and compressibility of the soil. In this study, the effect of variable permeability and compressibility on the consolidation behavior is investigated. For this objective, mathematical treatment of the behavior is presented. Subsequently, laboratory consolidation tests with mid-plane pore pressure measurements are conducted on soft, remolded, pre-consolidated and undisturbed samples of clay. The test results, when compared with the theoretical findings, indicate that most of the inherent discrepancies may be explained via the use of the theory developed in this study.

Keywords: Variable permeability, Compressibility, Soil consolidation

1 Introduction

Consolidation is a process by which soils decrease in volume. It occurs when stress is applied to a soil that causes the soil particles to pack together more tightly, therefore reducing its bulk volume. When this occurs in a soil that is saturated with water, water will be squeezed out of the soil. The theory of consolidation proposed by Terzaghi [1] is a very useful tool for the determination of settlement rates and amounts. Since the proposal of this theory, various researchers have investigated its validity and applicability. These subsequent studies have led to the development of various procedures for estimating settlements. Seed [2] discusses such various methods and procedures. The experience obtained through the years after the proposal of the Terzagi theory indicate that for one dimensional consolidation in particular, it gives results of acceptable accuracy in many field cases. However, laboratory tests on various types of clay indicate that, although the

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hydrodynamic approach presented by the theory cannot be disputed on the whole, there seem to be discrepancies between the theoretical predications and observations of the consolidation behavior. During the stage of primary consolidation, these apparent discrepancies are largely attributed to secondary (creep) effects and attempts have been made either to modify the assumptions implicit in Terzaghi's theory to agree more closely with observed behavior (Barden and Schiffman [3-4]) or to propose rheologic models which would better suit the observed behavior (Wahls and Lo [5-6]).

In this study, the main point of argument is that, although the existence of secondary effects may not be ignored, most of the discrepancies between the predictions based on Terzagi's theory and observed behavior during laboratory testing may be accounted for by modifying two assumption of constant permeability and the assumption of constant compressibility.

In order to carry the discussion further, a qualitative outline of the Terzaghi theory with its assumptions is accounted for in the following section.

The classical prediction procedure of the rate and amount of consolidation via Terzaghi's theory (1924) includes following assumption (Lambe [7]):

1. The soil is homogenous (uniform in composition throughout).

2. The soil is fully saturated (zero air voids due to water content being so high).

3. The solid particles and water are incompressible.

4. Compression and flow are one-dimensional (vertical axis being the one of interest).

5. Strains in the soil are relatively small.

6. Darcy's Law is valid for all hydraulic gradients.

7. The coefficient of permeability and the coefficient of volume compressibility remain constant throughout the process.

8. There is a unique relationship, independent of time, between the void ratio and effective stress.

If pore pressure dissipation measurements are also made during consolidation testing; it is possible to estimate the rate of settlement by making use of the dissipation time data. The usual consolidation coefficients calculated by two different procedures usually yield similar results for soft soils (Crawford [8]).

The conventional Terzaghi theory [1] proposed for fully saturated soils contains two assumptions which may be criticized from the view point of soil behavior. That is; the supposition of a constant compressibility coefficient a_v and a constant permeability coefficient *k*. It should be evident that as the consolidation proceeds, (effective stress increase) the soil attains a more compact structure which should inevitably result in a decrease in its overall compressibility. Evidence of this behavior has been obtained through several studies (Leonards et al. [9]). In addition, it has definitely been established that the permeability is a function of void ratio. It is obvious that the void ratio of a soil sample decreases during consolidation; therefore it is natural to expect a decreasing permeability coefficient during the process. In fact, otherresearches give experimental as well as theoretical evidence towards the recognition of a variable permeability (Barden [10], Schiffman [4], and Schmid [11]).

The propose of this study is to incorporate these variables in a mathematical treatment of the problem and to demonstrate by proper testing that the inclusion of these two variable factors may in fact account for most of the deviations that repeatedly occur between the predictions based on the Terzaghi theory and the test results (Lo [6], Crawford [8], Leonards [12]).

It should be noted that a similar problem has been treated by Barden and Berry [10] with a different mathematical approach which results in a non linear partial differential equation whose solution is obtained by a finite difference approach employing a suitable computer program, since a closed form solution cannot be obtained. Lekha et al.[13] studied consolidation of clays for variable permeability and compressibility and they presented a more generalized theory for vertical consolidation of a compressible medium of finite thickness, subjected to suddenly applied loading, assuming small strain and no creep. Their theory assumes small deformations, and thus the settlement is governed by vertical strains generated by an increment of loading, neglecting the effect of self-weight of the soil. Their solution takes into account e-log K and e-log σ' linear responses under instantaneous loading with Cc as the slope of the e-log σ' line and M the slope of the e-log K line, a parameter Cc/M is identified which is found to govern the rate of consolidation. Geng et al [14] studied non-linear consolidation of soil with variable compressibility and permeability under cyclic loadings. In their paper, a simple semi-analytical method has been developed to solve the one-dimensional non-linear consolidation problems by considering the changes of compressibility and permeability of the soil layer, subjected to complicated time-dependent cyclic loadings at the ground surface. Abbasi et al. [15] developed a finite difference approach for consolidation with variable compressibility, permeability and coefficient of consolidation. A spectral method is presented by Walker et al. [16] for analysis of vertical and radial consolidation in multilayered soil with PVDs assuming constant soil properties within each layer. Xie et al. [17] presented a solution for one dimensional consolidation of clayey soil with threshold gradient.

The line of treatment herein, on the other hand, arrives at the description of the consolidation process by a linear partial differential equation whose closed form solution is obtainable via the theory of linear partial differential equation.

2 Mathematical Development

In the mathematical treatment of the problem, the first problem is to decide on the nature of a functional relationship between permeability, compressibility and the main variables governing the process of consolidation. The equation is the manner in which these parameters are to be included into the mathematical model of the consolidation process, while retaining the other assumptions inherent in the classical Terzaghi theory.

Since the dependency of these parameters on void ratio is evident, the most reasonable approach would be to express them as functions of void ratio, or, since ratio is a function of pore water pressure, as functions of pore water pressures. Thus

$$k = k(u)$$
 and $a_v = a_v(u)$ (1)

Eq. (1) suggests that the properties are functions of both time and space. That is

$$k = k(z,t)$$
 and $a_v = a_v(z,t)$ (2)

At this stage, a postulate must be made as to the variation of the permeability and compressibility defined by eq. (2).

To illustrate the foundations of this postulate, reference is made to Fig. 1a. Prior to loading, the values of permeability and compressibility are constant with depth and may be denoted as k_0 and a_0 , respectively. As soon as the load is applied, consolidation starts and after an infinitesimal time, the excess pore water pressure on the drainage surface z = 0 become zero. Via eq. (2) this means that both permeability and compressibility reach their final values and remain constant thereafter at the surface. On the other hand, the values of these properties at any depth vary with time as consolidation proceeds. Therefore, at mid plane z = H via eq. (2), the permeability and compressibility are given respectively by

$$k_m = k(H,t)$$
 and $a_m = a_v(H,t)$ (3)

It is possible to describe the variation of these properties with space between drainage surface z = 0 and the mid plane z = H and to define the time dependent functions indicated in eq. (3). The variation of these properties with depth may be described by various mathematical functions. On the other hand, it is possible to define the "space averages" of permeability and of compressibility as follows:

$$\overline{k} = \frac{\int_{0}^{H} k(z,t) dz}{\int_{0}^{H} dz} \quad \text{and} \quad \overline{a} = \frac{\int_{0}^{H} a_{v}(z,t) dz}{\int_{0}^{H} dz}$$
(4)

At this stage, the variation of \overline{k} and \overline{a} with time during the consolidation process needs to be defined.

It seems feasible to define these relationships as functions of decay, i.e.

$$\overline{k} = k_i e^{-\alpha t}$$
 and $\overline{a} = a_i e^{-\beta t}$ (5)

With

$$\overline{k}(0) = k_i \quad \overline{a}(0) = a_i$$

$$\overline{k}(t_f) = k_f \quad \overline{a}(t_f) = a_f$$
(6)

In the expressions above k_i and a_i are the initial values of \overline{k} and \overline{a} , k_f and a_f are their final values, respectively, after a suitably long time t_f during which the primary consolidation is assumed to be almost complete. It is also possible to define the space variation of k and a_v with suitable functions of depth and carry on with the mathematical treatment by substituting these relationships in eqs. (4). Herein, it is assumed that the time t_f is long enough so that, although its value may be accepted as a finite value mathematically, the excess pore water pressure may be considered to be dissipated at the end of this period for all practical purposes. The final expression governing the dissipation of excess pore pressures by the former approach and by the analysis given herein are found to be substantially the same although the former one may be considered more "exact" by the mathematician. However to the benefit of this exactness, it entails the use of cumbersome mathematical formulations and some necessary simplifying assumptions derived from possible behavior of soils during the consolidation process to facilitate the analysis.

Reference to Fig 1b.shows the variation of compressibility via effective stress (or with time for all practical purposes) and represents a general curve usually obtained through consolidation dissipation tests.



Figure 1: (a) Variation of pore water pressure, permeability and compressibility and (b) Variation of Compressibility during consolidation

Eqs. (5) mean that both \overline{a} and \overline{k} vary with time as a function of decay and reach their final values at the end of consolidation.

Once the mathematical formulation of permeability \overline{k} and \overline{a} (eq. 5) are made, it remains to write down the continuity equation of consolidation in the usual manner and substituting these mathematical formulations therein, to obtain the governing equation of consolidation.

The continuity expression is written as follows:

$$\frac{\partial}{\partial z} \left(\bar{k} \frac{\partial u}{\partial z} \right) = \frac{\gamma_w}{1 + e_0} \times \frac{\partial e}{\partial t}$$
(7)

On the other hand, the effective stress law gives:

$$\overline{\sigma} = \sigma - u \tag{8}$$

where $\overline{\sigma}$ = effective stress, σ = total stress, u = pore pressure.

The compressibility is defined as

$$\overline{a} = -\frac{\partial e}{\partial \overline{\sigma}} = \frac{\partial e}{\partial u} \tag{9}$$

Now, remembering that $\frac{\partial e}{\partial t} = \frac{\partial e}{\partial u} \times \frac{\partial u}{\partial t}$, and substituting eq.(9) in eq. (7), one obtains:

$$\frac{\partial}{\partial z} \left(\bar{k} \frac{\partial u}{\partial z} \right) = \frac{\gamma_w}{1 + e_0} \times \bar{a} \times \frac{\partial u}{\partial t}$$
(10)

In eq (10) \overline{k} is independent of space, therefore this equation can be written in the following form:

$$\frac{\partial^2 u}{\partial z^2} = \frac{\gamma_w}{1 + e_0} \times \frac{\overline{a}}{\overline{k}} \times \frac{\partial u}{\partial t}$$
(11)

Substituting the values of \overline{a} and \overline{k} from eq (5) into this expression; the governing partial differential equation is obtained:

$$\frac{\partial^2 u}{\partial z^2} = \frac{\gamma_w a_i}{(1+e_0)k_i} e^{(\alpha-\beta)} \frac{\partial u}{\partial t}$$
(12)

It is possible to express this equation in a dimensionless form by specifying the variables W and Z

$$W = \frac{u}{u_0} \quad , \quad Z = \frac{z}{H} \tag{13}$$

where u_0 = initial pore pressure, H = characteristic thickness and a time factor T such that

$$T = \frac{(1+e_0)k_i}{\gamma_w a_i H^2} t \tag{14}$$

Also specifying a constant A,

 $A = \frac{(\alpha - \beta)H^2}{c_v}$ (15)
(16)

the usual coefficient of consolidation, eq.(12) becomes:

$$\frac{\partial^2 W}{\partial Z^2} = e^{AT} \frac{\partial W}{\partial T}$$
(17)

It is obvious that depending on the relative values of α and β (signifying the effects of permeability and compressibility respectively) A may assume both positive and negative values; therefore, eq. (17) needs to be solved for both positive and negative possible values of A. It is of further interest to note that in the case of constant permeability and compressibility during the process, or if the rate of change of both parameters is the same, A becomes equal to zero and the process is governed by the partial differential equation arrived at by Terzaghi [1].

$$\frac{\partial^2 W}{\partial Z^2} = \frac{\partial W}{\partial T} \tag{18}$$

The solution of eq. (17), subject to the usual oedemeter boundary conditions, is obtained as:

$$W(Z,T) = \sum_{n=0}^{\infty} \left\{ \frac{4}{(2n+1)\pi} \left(\sin \frac{2n+1}{2} \pi Z \right) \varepsilon^{-\frac{\pi^2 (2n+1)^2}{4} \times \frac{e^{\pm AT} - 1}{\pm A}} \right\}$$
(19)

where ε is the naperian base of the logarithm and n is an integer. Similarly, consolidation is defined by the expression:

$$U = 1 - \frac{\int_{0}^{1} W(Z,T) dZ}{\int_{0}^{1} W(Z,0) dZ} = 1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \varepsilon^{-\frac{\pi^2 (2n+1)^2}{4} \times \frac{\varepsilon^{\pm AT} - 1}{\pm A}}$$
(20)

and the expression for mid plane pore pressure is given by the expression:

$$W_{m}(Z,T) = \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin \frac{2n+1}{2} \pi \varepsilon^{-\frac{\pi^{2}(2n+1)^{2}}{4} \times \frac{\varepsilon^{\pm AT}-1}{4}}$$
(21)

Eqs. (20) and (21) are plotted for various positive and negative values of the parameter A as illustrated in Fig. 2 and Fig.3, respectively.

As in the case of the classical theory of consolidation, these curves constitute the bases of the evaluation of rates of settlement via the theory developed in this paper. The use of these curves may often involve a trial and error procedure with regard to the appropriate selection of the value of A.



Figure2: Percent consolidation



Figure 3: Mid-Plane pore pressure dissipation

3 Experimental Investigation

The soil used in the study is the grey Clay. The properties of these clays are related in general. The particular soil used showed the following index properties: LL = 68% PL = 23% PI = 45% SL = 12% with a Casagrande classification of CH. It was intended to study the consolidation of the soil behavior in three distinct conditions these being:

a) Soil sample denoted by S - 1:

Soil in a remolded state at a soft consistency. Duely for this state the first soil sample was prepared at a consistency equaling a water content equal to LL-10%

b) Soil sample denoted by S - 2:

Soil initially in a soft consistency, however, in a pre-consolidated state. For this purpose the soil sample in (a) was consolidated up to a certain effective stress, rebound, and then tested to observe its behavior. Thus, the soil was tested in a laboratory induced pre-consolidated state.

c) Soil sample denoted by S - 3:

Soil in its natural state, being soaked prior to testing without allowing any change in volume. This procedure may also be called the soak swell prevented type of test which is used for expansive Clay. Equipment and testing procedure: The equipment consists of a consolidometer in which base pore pressure could be measured by means of an automatic non flow type of pore pressure apparatus. The layout of the equipment is schematically illustrated in Fig.4.

Oedometer used, is made of steel with inside diameter of 102.6 mm and height of 25.4 mm. Details of the oedometer is shown in Fig. 4. At the bottom a porous stone was housed in order to measure mid plane pore pressures. The porous stone at the top of the sample is for drainage of water as the consolidation proceedings.

Application of the vertical stress was provided by a steel sphere which was placed on the steel circular plate. One dimensional vertical drainage was provided by porous stone and a top drain valve.

Settlements were followed by a dial gage placed on the plate where oedometer was put. The mid plane pore pressure was measured at a ceramic disc placed in the center of the base, using pore pressure apparatus which was de-aired and periodically checked by the parts of the oedometer are shown in Fig. 4.



Figure 4: Pore Pressure Device, Oedometer

Sample S-1 is loaded under the following increments, the compression and pore pressure development dissipation being measured for 24 hours under each increment (in $\frac{kg}{cm^2}$): 0.00-0.25; 0.50; 1.0; 2.0; 4.0; and 8.0. $\frac{kg}{cm^2}$): 0.00-0.25; 0.50; 1.0; 2.0; 4.0; and 8.0. The pore pressure measurement line is then closed and the sample unloaded to 2.00 kg/cm^2 . This load is kept on the sample for 24 hours. Thus, the new loading stage is made on sample S-2 which is preconsolidated to $8.0 \frac{kg}{cm^2}$. This loading stage consisted of reloading sample S-2 thus prepared in two increments, namely: 2.0-4.0; 4.0-8.0 $\frac{kg}{cm^2}$. For these two increments the compression and pore pressure data are observed as in sample S-1. The sample in its original void ratio, (sample S-3) is flooded without allowing any volume change for 24 hours. Then, it is loaded in the following increments: 1.0; 2.0; 4.0; 8.0 $\frac{kg}{cm^2}$. During loading, the necessary data is obtained as in the previous cases. It should be noted at this stage that the pressure increment ratio used throughout testing is 1.00 to minimize the secondary time effects. On the other hand, another important factor affecting consolidation behavior is the preconsolidation pressure. In this investigation, it is hoped to throw some light on to this controversial point by means of the behavior of test sample S-2, at least for the soil investigated.

4 Presentation of Results

In presenting the results, the very first step would be the determination of the amount of primary consolidation. Where pore pressure measurements are made, it is better to use the criterion of zero excess pore empirical procedures of curve fitting. Therefore, as was proposed by Crawford [8], for each increment, compression amounts were plotted versus the pore pressure dissipation and the straight line portion of the curve (which is straight up to about 70% consolidation as predicted by means of the mid plane pore pressure data) was extrapolated to zero pore pressures to determine the amount of compression during primary consolidation. This amount is designated as d_{100} . Therefore, if compression of the sample at any time is d_r , the percent consolidation U at that time is found by the expression,

$$U\% = \frac{d_t}{d_{100}} \times 100$$
 (22)

Using this expression, the percent consolidation values are calculated for each increment and plotted against time in the lower portion in Fig 5 to 16 inclusive. They are shown by the solid lines marked "experimental".

Subsequently, using the time values corresponding to 50% consolidation on these curves, the coefficient of consolidation of consolidation C_v is obtained via eq.(16) for the

Terzaghi case (Fig.2, A = 0) and thus, these curves are fitted through 50% consolidation by the Terzaghi predictions. These predictions are shown in the figures by the dotted lines marked "Terzaghi"

The method proposed herein was then applied as follows:

By inspection a suitable A value is chosen and using the time value corresponding to 50% consolidation once again, a new consolidation coefficient corresponding to this A value is found. Then the fitting procedure related above is carried out using the theoretically developed curves for the chosen A value in Fig. 2. This trial procedure was repeated until a good agreement between the "experimental" and "predicted" curves was obtained. The curves that fit the actual behavior in the best manner are also shown on the same on the same figure as above. The same fitting procedure for the appropriate A value found as above is applied to the observed pore pressure dissipation values for comparison, this time making use of the theoretically developed curves in Fig. 3. The results of these comparisons are presented in the top portions of Figs. 5 to 16 inclusive, solid lines again representing the experimental observations, and the other corresponding to the fitting made via the relevant A value.

5 Discussion of Results

An analysis of the curves obtained by the procedures related in the previous section may be related as follows:

In general, it is obvious that for the soil investigated in various states, the Terzaghi theory seems to be still a very powerful tool in predicting the rates of settlement. However, except for the three pressure increments of sample S - 1, whose behavior is presented by Figs 6, 7, 8 and for the last pressure increment of sample S - 3 presented by Fig. 16, there exist discrepancies between the actual the actual consolidation behavior and its Terzaghi predictions. These deviations, which are largely attributed to "secondary effects", are seen to be correctable by means of the theory forwarded in this study. This fact very strongly supports the idea that these deviations mostly result from varying compressibility and permeability during consolidation, which is the starting point of the theoretical development in this investigation. The cases exemplified by Figs 6, 7, 8 and 16, closely agree with Terzaghi behavior A = 0. The actual pore pressure dissipation behavior, where fitted either by the Terzaghi theory A = 0 or the theory proposed herein (A = appropriate value) seems to be very closely predictable. This shows once again the predominate character of the hydrodynamic process during consolidation rather than the secondary effects.

It is worth nothing that the parameter A is always positive. Eq. (15) indicates that in this case, the rate of decrease of permeability is the predominant factor rather than the rate of decrease of compressibility, for the soils tested. It is also interesting to note that for the preconsolidated sample S - 2 this generalized theory is applicable for determining the rates of compression, since the applicability of the Terzaghi theory (rather the hydrodynamic philosophy behind it) has been questioned for preconsolidated soils.

Reviewing the behavior of the sample of soft consistency (sample S-1) it is seen that deviations from Terzaghi theory occur when the first load increment is applied (Fig. 5) and again when increments of large magnitude are applied (Figs 9, 10, 11). This may be

due to the fact that in both cases the soil sample is presumably subjected to larger alteration in its structure and its engineering properties during consolidation. In fact, both compressibility and permeability should be considered as functions of the magnitude of applied pressure as well as an intrinsic property of the soil depending on its void ratio, structure, degree of saturation, etc. Therefore, it would not be wrong to presume that a soil sample loaded in increments up to a certain pressure would follow a different pressure deformation curve than if the ultimate pressure had been applied all in one step.

In this investigation, note should be made of the fact that that the usual empirical curve fitting methods such as the square root fitting method or the logarithmic fitting method are not employed, mainly due to the fact that the measurement of pore pressure is believed to be a better substitute. In view of the present study on the other hand, a criticism of these methods may be made. Fig. 17 shows the average consolidation plotted against the square root of the time factor, for the Terzaghi case and for values of A = +1.00 and A = -1.00 it is obvious that the application of the square root fitting method to any soil behavior in any other manner than that of Terzaghi A = 0 would give vastly incorrect results both as to the time of completion of primary consolidation and to the value of the coefficient of consolidation C_v .

Fig.18 shows the same curves plotted against the logarithm of time. It seems from these figures that although deviations are apparent, the errors introduced by using the "logarithm of time" fitting method would be smaller. For A = +1.00 this method is seen to yield about 90% primary consolidation instead of 100%. For larger positive A = +1.00 values, the errors become much larger. This observation may in fact account at least partly for the discrepancies that occur between the settlement rates predicted in the laboratory by these empirical rules and those actually taking place in the field.

6 Conclusions

As a result of the present study the following conclusions may be reached:

1) The Terzagi theory, in predicting the settlement rates is a very valuable tool.

2) The observed departures from this theory seem to be mostly due to the variation in the compressibility and the permeability of the soil. For the specific soil tested, permeability seems to be the predominant factor.

3) Using the theoretical treatment forwarded in this study, it is possible to eliminate largely the discrepancies and predict the rates of settlement more accurately.

4) As far as the soil used in this study, it is shown that the proposed theory may also account for its apparent departures from the Terzaghi behavior in a pre-consolidated state as well.

5) The empirical curve fitting procedures should be applied with caution. Although the logarithmic fitting procedure seems to be more reliable, for important civil engineering estimates of rate of settlement and for research, pore pressure dissipation tests seem to be the best procedure.



Figure5 : Dissipation Consolidation versus Time







Figure 7: Dissipation Consolidation versus Time







Figure9: Dissipation Consolidation versus Time



Figure10:Dissipation Consolidation versus Time



Figure11: Dissipation Consolidation versus Time



Figure12: Dissipation Consolidation versus Time



Figure13: Dissipation Consolidation versus Time



Figure14: Dissipation Consolidation versus Time



Figure15: Dissipation Consolidation versus Time



Figure16: Dissipation Consolidation versus Time



Figure 17: Consolidation (%) versus (Square root scale)



Figure18: Consolidation versus Logarithm of Time Factor

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