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Asian Options Greeks with Heston Stochastic Model Parameters

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Abstract

An Asian option is an example of exotic options. Its payoff depends on the average of the underlying asset prices. In this paper we focused on analytical approximations and a study of sensitivities (Greeks) of Asian options with Heston stochastic volatility model parameters, after a brief introduction to the Black-Scholes theory. Only fixed strike Asian options is considered. After a study of Greeks with Heston model parameter, a comparison of some approximated Greeks against those obtained previously with different approaches is also done.
This study is conducted to provide some knowledge and application about the Greeks.

Keywords: Asian options; Greeks; Approximations; Heston model; Parameters

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1 Introduction

Asian options are securities with payoff which depends on the average of the underlying stock price over certain time interval. The average may be over the entire time period between initiation and expiration or may be over some period of time that begins later than the initiation of the option and ends with the options expiration. Asian options are hard to price each analytically and numerically. These options are popular in financial markets where the averaging feature introduces a smoothing effect. The Greeks are a collection of statistical values that degree the danger involved in an alternatives agreement in relation to sure underlying variables. They play an essential position in hedging and risk management. It requires a good understanding of their properties and qualitative behavior under changes in the model parameters. For this reason, a exquisite deal of effort has been put into developing precise computational methods for numerical evaluation of sensitivities. Most of these methods are: the Monte Carlo method which is very popular because of its flexibility and ease of implementation and Finite difference methods also provide a very flexible and efficient for pricing Asian options [11]. The pathwise method which is one the most effective and is based on a technique generally called infinitesimal perturbation analysis [4]. And the Malliavin calculus methods are also known to be more efficient and improve the speed of the Greeks calculation [8].

Compared to standard vanilla options, Asian options have some obvious advantages. First of all, they are often cheaper and better suited for hedging purposes. Secondly, Asian options reduce the risk of price manipulation near the maturity date,when the underlying is a thinly traded asset or commodity. But their Greeks are more challenging to price, due to the absence of analytical expressions for their prices. [17] gave the analytical approximations for the Greeks of an Asian option in the Black Scholes model when the equivalent implied volatility was obtained by the large deviation theory. The aim of the existing paper is to study the sensitivities of the Asian options under the assumption that the equivalent implied volatility is a function of the Heston stochastic volatility model parameters.We obtain analytical approximate prices of Asian options in the case of the Black Scholes model. The pricing of Asian options with continuous-time averaging in this model. We just consider the case of European style Asian options and the fixe strike Asian options.

The academic literature proposes several methods that accommodate the path dependency of Asian options. [22] proposed numerical approximations, and Monte Carlo simulations were employed by [14] and also [3]. A great variety of numerical and exact methods have been proposed for their pricing by [20], [7] and [10]. Most of these methods are numerically and computationally intensive. We quote [23], who proposed a semi-analytical method for pricing and hedging continuously sampled Asian options. [1] discus the problem of approximating the price of options on discrete and continuous arithmetic averages of the underlying, i.e., discretely and continuously monitored Asian options, in local volatility models. [9] also discussed Pricing Asian Options with Stochastic Volatility and he use the fast mean-reverting stochastic volatility asymptotic analysis to derive an approximation of the option price which takes into account the skew of the implied volatility surface. Furthermore, [18] as discussed about a study of the short maturity asymptotic for Asian options with continuous-time averaging, under the assumption that the underlying asset follows the Constant Elasticity of Variance (CEV) model. In the general case, mean-reversion is considered to be an important feature of observed volatility, and thus all plausible models are of the Ornstein-Uhlenbeck type [13]. Many of researchers model the variance using a square root process, see [21] and [12]. Monte Carlo simulation can be used to generate an unbiased estimator of the price of the derivative securities [5]. [19] have discussed Computing Option Price Sensitivities Using Homogeneity and Other Tricks and have shown that most remarkably some relations of the Greeks are based on properties of the normal distribution refreshing the active interplay between mathematics and financial markets. The derivatives of Asian options call options prices has been discussed wherein an integral forms for key quantities in the price of Asian option and its derivatives were presented see [6]. However, a study of sensitivities of Asian options in the Black-Scholes model has been done by [17], using using large deviations theory and following from a small maturity/volatility approximation for the option prices.

We now give a short summary of the rest of the paper. In the next section we introduce the Black Scholes model and the Heston stochastic volatility model. Section 3 presents some Analytical Approximate formula for the Asian Greeks, and present some new results for Vanna and Volga of the fixed strike

Asian options. In section 4 we implement a series of numerical tests and discuss the behavior of these Greeks of Asian options with respect to some Heston parameters. And finally, Section 5 provides a short summary of the paper.

2 Models Presentation

2.1 Black-Scholes Model and Option Pricing

We work on a probability space $(\Omega, \mathbb{P}, \mathcal{F})$ with a filtration $(\mathcal{F}_t)_{t\geq 0}$ supporting two Brownian motions and satisfying the usual conditions. In 1973, Fischer Black and Myron Scholes published their ground-breaking paper [2]. [16] adjusted the Black Scholes formula to enable it to price European options on stocks or stock indices paying a known dividend yield.

The Black-Scholes model assumes a market consisting of a single risky asset and a risk-free bank account. This market is given by the equations.

$$\frac{dS_t}{S_t} = (r - q)\, dt + \sigma dW_t \tag{1}$$

Where W_t is Brownian motion under risk neutral measure and the interpretation of the parameters is as follows: r is the risk free rate,$\sigma > 0$ is the volatility of the risky asset,q is dividend.

Let T be the maturity and K be the fixed-strike price. The price of the fixed-strike call and put Asian options are given by

$$C\,(K, T, \sigma, r, q) = e^{-rT} E\left[\max\{\frac{1}{T}\int_0^T S_t dt - K, 0\}\right], \tag{2}$$

$$P\,(K, T, \sigma, r, q) = e^{-rT} E\left[\max\{K - \frac{1}{T}\int_0^T S_t dt, 0\}\right], \tag{3}$$

The approximation has been proposed in [17] for the Asian option prices has a form similar to the Black Scholes formula.

2.2 Heston Stochastic Volatility Model

The Black-Scholes model approximately describes the behaviour of underlying asset prices and provides a convenient closed-form formula for option prices. It provides an important benchmark to evaluate the performance of other models. However, [12] assumes that the process S_t follows a log-normal distribution, and the process v_t follows a Cox-Ingersoll-Ross process (CIR) process (1985). The model is given as:

$$\frac{dS_t}{S_t} = (r - q)\, dt + \sqrt{v_t} dW_t \tag{4}$$

$$dv_t = k\,(\theta - v_t)\, dt + \sigma \sqrt{v_t} dZ_t \tag{5}$$

$$dW_t dZ_t = \rho dt \tag{6}$$

• r is the constant risk-free rate
• q is the dividend yield
• θ the mean reversion level for the variance
• k the mean reversion speed for the variance
• σ is the volatility of volatility
• S_t anb v_t are the price and volatility process respectively
• To take into account the leverage effect, stock returns and implied volatility are negatively correlated,dW_t and dZ_t are correlated Wiener process, and the correlation coefficients is $\rho \in [-1, 1]$.

Note that $2k\theta > \sigma^2$ ensures that zero is an unattainable boundary for the process v_t.

3 Analytical approximate formulas of Asian option Greeks

Theorem 3.1. *Consider the Cox-Ingersoll* (*CIR*) *interest rate Model.*

$$dv_t = k\,(\theta - v_t)\, dt + \sigma \sqrt{v_t} dZ_t \tag{7}$$

Then the exact solution is

$$v_t = v_0 e^{-kt} + \theta \left(1 - e^{-kt}\right) + \sigma e^{-kt} \int_0^t e^{ku} \sqrt{v_u} dZ_u \tag{8}$$

According the theorem of the CIR model has no general explicit solution. Denote the mean and the variance, respectively, $\mu_v = E\,(v_T)$ and

$\sigma_v^2 = Var\left(v_T\right)$
where,

$$E\left(v_T\right) = v_0 e^{-kT} + \theta\left(1 - e^{-kT}\right) \tag{9}$$

$$Var\left(v_T\right) = k v_0 \theta\left(e^{-kT} - e^{-2kT}\right) + \frac{\theta\sigma^2}{2k}\left(1 - e^{-kT} + e^{-2kT}\right) \tag{10}$$

The analytical approximations in the Black-Scholes framework to a Call and Put options which are simmilary to the those given by [18] but here the equivalent implied volatility is depended on time and it also depends on the Heston stochastic volatility model parameters. these approximations are given as

$$C\left(K, T, \sigma, r, q\right) \approx e^{-rT}\left[F(T)N(d_1) - KN(d_2)\right] \tag{11}$$

$$P\left(K, T, \sigma, r, q\right) \approx e^{-rT}\left[KN(-d_2) - F(T)N(-d_1)\right] \tag{12}$$

where, $N(.)$ is the cumulative function of the Normal distribution.

$$F(T) = S_0 \frac{e^{(r-q)T} - 1}{(r-q)T} \tag{13}$$

$$d_1 = \frac{\ln(\frac{F(T)}{K}) + \frac{\sigma_A^2 T}{2}}{\sigma_A \sqrt{T}} \tag{14}$$

$$d_2 = d_1 - \sigma_A \sqrt{T} \tag{15}$$

and the implied volatility is a function not only of the explicit parameters, namely, S, v and T, but is also a function of all the implicit ones, the strike K,the interest rate r, and the dividend yield q. According to [15] on page 129, the implied volatility can be approximated as,

$$\sigma_A(x) \approx \mu_v + \frac{1}{4}\frac{\sigma_v^2}{\mu_v^2}\left(\frac{x^2}{T} - \mu_v - \frac{1}{4}\mu_v^2 T\right) \tag{16}$$

where, $x = \log\left(\frac{K}{S_0}\right) + \left(r - q\right)T$

Remark 3.2. *1. The Call Asian option is Out-of-the-money if $K > F(T)$ and when $K < F(T)$, the Put Asian option is Out-of-the-money*

2. The prices of Call and Put Asian options are related by the Put-Call Parity given as

$$C\left(K, T, \sigma, r, q\right) - P\left(K, T, \sigma, r, q\right) = e^{-rT}\left[F(T) - K\right] \tag{17}$$

Using the short maturity approximation (11), (12) one can derive simple approximations for the Greeks of the fixed-strike Asian options. However, numerical testing will show that they give reasonably good approximations for these sensitiv- ities with those Heston Model Parameters.

Delta

The Delta of the approximated Asian call/put option is defined as

$$\Delta_{Call} = \left(\frac{\partial C\left(K, T, \sigma, r, q\right)}{\partial S_0} \right), \Delta_{Put} = \left(\frac{\partial P\left(K, T, \sigma, r, q\right)}{\partial S_0} \right) \tag{18}$$

Using 11 we the get the explicit result as

$$\Delta_{Call} = e^{-rT} \frac{F(T)}{S_0} \{N(d_1) - \sqrt{T} n(d_1)(\sigma_A)'\} \tag{19}$$

where, $n(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$ is the density function of the Normal distribution. $(\sigma_A)' = \frac{\partial}{\partial x} \sigma_A$

Then from equation (16) we get,

$$(\sigma_A)' = \frac{1}{2T} \frac{\sigma_v^2}{\mu_v^2} x \tag{20}$$

Now from the Put-call parity (17) we get Delta Put,

$$\Delta_{Put} = \Delta_{Call} - \frac{e^{-rT}}{(r-q)T} \{e^{(r-q)T} - 1\} \tag{21}$$

Gamma

The Gamma approximated formulas of Asian call/put option are obtained by using these following definitions,

$$\Gamma_{Call} = \frac{\partial \Delta_{Call}}{\partial S}, \Gamma_{Put} = \frac{\partial \Delta_{Put}}{\partial S} \tag{22}$$

$$\Gamma_{Call} = e^{-rT} \frac{F(T)}{S_0^2} \frac{1}{\sigma_A \sqrt{T}} n(d_1) \left[\left(1 + \sqrt{T}(\sigma_A)'\right) \left(1 + \frac{(\sigma_A)'}{\sigma_A} \log \frac{F(T)}{S_0}\right) + \sigma_A (\sigma_A)'' T \right] \tag{23}$$

where,

$$(\sigma_A)'' = \frac{1}{2T} \frac{\sigma_v^2}{\mu_v^2} \tag{24}$$

Furthermore, the Gamma Put is equal to the Gamma call from equation 21

$$\Gamma_{Put} = \Gamma_{Call}$$

Vega

Vega is the first derivative of option price with respect to the implied volatility. Since the implied volatility is dependent on strike and maturity, Vega is also a function of strike and maturity. Therefore, strictly speaking, Vega is a risk sensitivity only defined in the Black-Scholes model. In stochastic volatility models, the constant volatility is displaced by the stochastic volatility whose process is characterized by a set of model parameters. The volatility risk in terms of Vega in the Black-Scholes model is then distributed to the corresponding model parameters of the stochastic volatility. Note that Vega expresses essentially the risks associated with the parallel change of the constant volatility, hence it makes sense to formulate a similar Greek associated with the parallel change of the volatility surface. In the Heston model, the spot value and the mean level of the stochastic volatility are responsible for the level of volatility dynamics. Hence, it is intuitive and reasonable in stochastic volatility models to define the so-called mean Vegas based in the spot volatility and the mean level (see [24]). But here, we are interested of the vega base on the spot. The Vega approximated Asian option Call based on the spot volatility is define as

$$v_{1,Call} = 2\sqrt{v_0}\frac{\partial C}{\partial v_0} \tag{25}$$

and is given by

$$v_{1,Call} = 2\sqrt{v_0}e^{-rT}F(T)n(d_1)\sqrt{T}\frac{\partial \sigma_A}{\partial v_0} \tag{26}$$

with,

$$\frac{\partial \sigma_A}{\partial v_0} = e^{-kT} + \frac{1}{2}\left(\frac{x^2}{T} - \mu_v - \frac{1}{4}\mu_v^2 T\right)\beta - \frac{\sigma_v^2}{4\mu_v^2}\left[1 + \frac{1}{2}\mu_v T\right]e^{-kT} \tag{27}$$

where,

$$\beta = \frac{k\theta\sigma_v\mu_v\left(e^{-kT} - e^{-2kT}\right) - \sigma_v^2 e^{-kT}}{\mu_v^2}$$

Vanna

The Vanna of the approximated Asian call option is defined as

$$Vanna = 2\sqrt{v_0}\frac{\partial^2 C}{\partial S_0 \partial v_0} \tag{28}$$

and is given by

$$Vanna = v_{1,Call}\frac{1}{S_0}\left[\left(\frac{1}{\sqrt{T}} + d_1\sigma'_A\right)\left[\frac{\frac{T}{2}\sigma_A^2 - \log\left(\frac{F(T)}{K}\right)}{\sqrt{T}\sigma_A^2}\right] - \frac{\partial\sigma'_A}{\partial v_0}/\frac{\partial\sigma_A}{\partial v_0}\right] \tag{29}$$

where,

$$\frac{\partial\sigma'_A}{\partial v_0} = \frac{x}{T}\beta \tag{30}$$

Volga

The Volga approximated Asian call option is defined as

$$Volga = \frac{\partial^2 C}{\partial v_0^2} \tag{31}$$

and is given as

$$volga = 2v_{1,call}\left[\frac{1}{2\sqrt{v_0}} - \sqrt{v_0}d_1\frac{\partial d_1}{\partial v_0} + \sqrt{v_0}\frac{\partial^2\sigma_A}{\partial v_0^2}/\frac{\partial\sigma_A}{\partial v_0}\right] \tag{32}$$

where,

$\frac{\partial^2\sigma_A}{\partial v_0^2} = \frac{\partial f}{\partial v_0}e^{-kT} + \frac{1}{2}\left[\frac{\partial\beta}{\partial v_0}h + \frac{\partial h}{\partial v_0}\beta\right]$

$\frac{\partial f}{\partial v_0} = -\frac{1}{2}\left[\left(1 + \frac{T}{2}\mu_v\right)\beta + \frac{T}{4}\frac{\sigma_v^2}{\mu_v^2}e^{-kT}\right]$

$\frac{\partial\beta}{\partial v_0} = k\theta\frac{\beta}{\sigma_v}\left(e^{-kT} - 2e^{-2kT}\right) - 2\frac{\beta}{\mu_v}e^{-kT}$

$\frac{\partial h}{\partial v_0} = -\left(1 + \frac{T}{2}\mu_v\right)e^{-kT}$

$f = 1 - \frac{1}{4}\frac{\sigma_v^2}{\mu_v^2}\left(1 + \frac{T}{2}\mu_v\right)$

$h = \frac{x^2}{T} - \mu_v - \frac{1}{4}\mu_v^2 T$

4 Numerical Tests

Let us fixe $T = 0.5$; $\sigma = 0.016$; $k = 5$; $\theta = 0.16$; $\rho = 0.01$; $r = 0.02$; $K = 100$; $S_0 = 100$; $v_0 = 0.0625$, $q = 0$. Next we change one of the variables at a time and see how the Asian Call option changes as this particular variable changes With the Heston model parameters.

Fisrt : Change the spot stock price from 80 to 130. Figure 1 shows that for a Call option, the price increases when the underlying price, S , increases as expected.

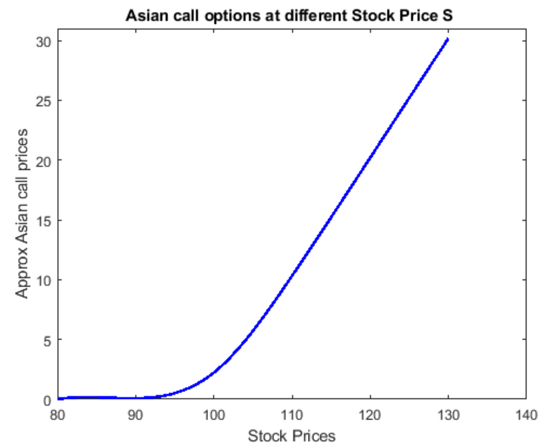

Figure 1: Asian Call Price under different Stock Prices

Second: Change the instantaneous variance v_0 from 0 to 0.5. Figure 2, Figure 3, Figure 4 respectively show the volatility square, v_0 , has a positive effect on the Asian option price, the larger the volatility, the higher the price. All three Figures have the same increasing trend indicating that the option prices increases as v_0 increases. ATM, ITM and OTM gives similar results in terms of Option price differences but in terms of relative difference it is OTM which is the most sensitive.

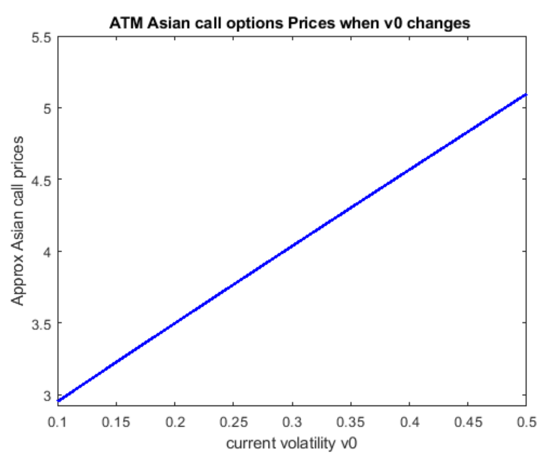

Figure 2: ATM Call Price under Different Instantaneous Volatility v_0

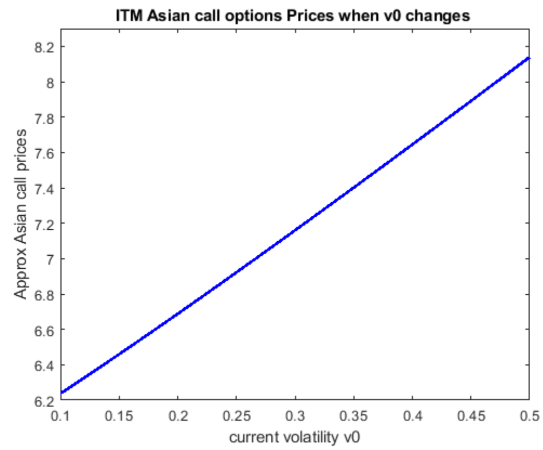

Figure 3: ITM Call Price under Different Instantaneous Volatility v_0

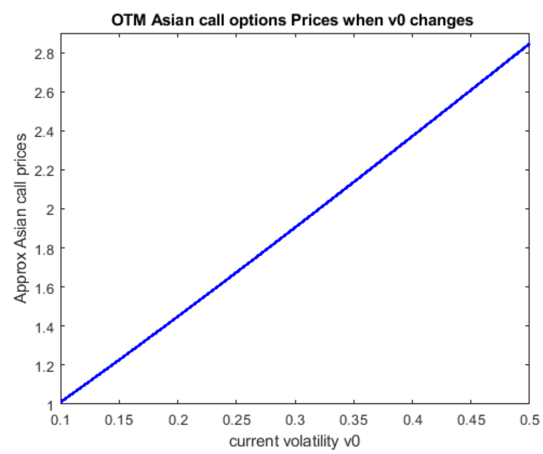

Figure 4: OTM Call Price under Different Instantaneous Volatility v_0

Consider an Asian Call option on an underlying $S_0 = 100$ with half of year to maturity; in order to study the implied volatility, different strike prices are required. The strike prices have to be chosen carefully to avoid too large or too small values. If the strike price is too high compared with the underlying price, the option would be deep ITM and fall into the early exercise region. On the other hand, if the strike is too low, the option would be deep OTM, which has little value. Either case could cause the failure of the option pricing model. Figure 5 is the implied volatility of an Asian options for $T = 0.5$; $\sigma = 0.016$; $k = 5$; $\theta = 0.16$; $\rho = 0.01$; $r = 0.02$; $S_0 = 100$; $v_0 = 0.0625$ with a number of different strike prices (K from 80 to 130). The pattern that implied volatility changes faster as the option goes from OTM to ITM is known in the literature as a "volatility sneer".

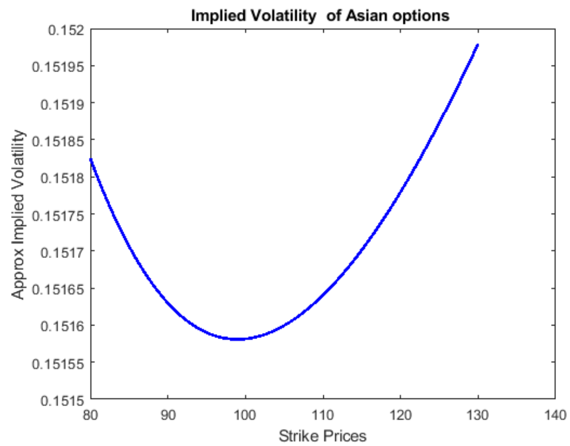

Figure 5: Implied Volatility of Asian Option with Heston Model parameters

Figure 6: shows that the delta of Asian call option is positive, which is to be expected, since an increase in the stock price would make the call worth more. A deep ITM call behaves as if one is long the underlying, and hence the corresponding delta is 1. A deep OTM call would have very little change in price as the underlying moves, hence the delta is 0. The range of delta for a call is $[0, 1]$.

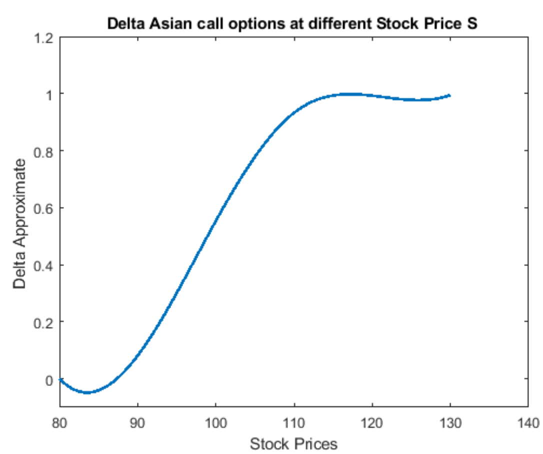

Figure 6: Delta of Asian Option with Heston Model parameters

Now let $K = 100$; $T = 0.5$; $\sigma = 0.016$; $k = 5$; $\theta = 0.1$; $\rho = 0.01$; $r = 0.02$;

$S_0 = 100$;and $v_0 = 0.0625$. Change one parameter at a time and hold others constant to assess the effect of the six structure parameters on option pricing (ATM study):
Each time, we change one parameter while holding other parameters constant. The frst columns contains the parameters to be changed. The second column is the value of the changing parameter. The third and forth columns are the Delta option values under each changed parameters with instantaneous volatility $v_0 = 0.02$ and $v_0 = 0.0625$ respectively.

Firstly Table 1 demonstrates that a large value of , θ and r leads to a higher values of Delta under different v_0. As we increase σ , and k the Delta values decreases.

Table 1: Delta Option with Heston Model Parameters

Parameters	Values	$v_0 = 0.02$	$v_0 = 0.0625$
k	3	0.55456540	0.55369519
	4	0.55385066	0.55355740
	5	0.55361246	0.55350922
θ	0.1	0.55361246	0.55350922
	0.16	0.55787409	0.55840788
	0.2	0.56434565	0.56502755
r	0.02	0.55361246	0.55350922
	0.1	0.65569015	0.65173781
	0.2	0.75796942	0.75117879
σ	0.016	0.55361246	0.55350922
	0.02	0.55361246	0.55350922
	0.3	0.55361206	0.55350808

Figure 7 shows that Gamma is greatest when the strike price is close to the stock price (ATM) and declines as the option moves away from the strike price and becomes further ITM or further OTM. The Approximate Gamma of an Asian option obtained is positive. As Figure 4.6 shows by considering the case of Asian Call option. As the underlying increases, we know that the delta increases, since it is more likely to be ITM. Hence, this tells us that Gamma, which is the rate of change of delta, is positive.

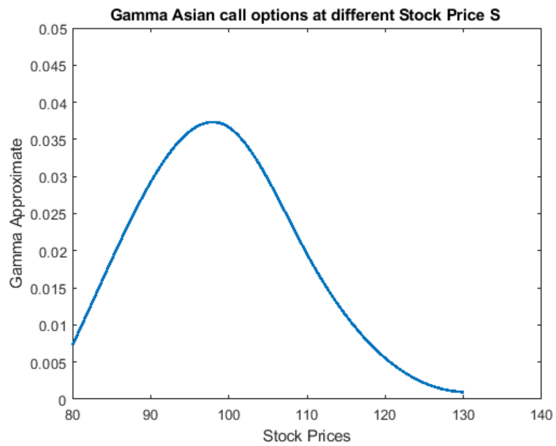

Figure 7: Gamma of Asian Option with Heston Model parameters

Secondly Table 2 demonstrates that a large value of k, θ and r leads to a lower values of Gamma under different v_0. As we increase σ , the Gamma values increases. And Also, a larger v_0 always results in a lower value of Gamma.

Table 2: Gamma Option with Heston Model Parameters

Parameters	Values	$v_0 = 0.02$	$v_0 = 0.0625$
	3	0.06763219	0.06067256
k	4	0.06232057	0.05856572
	5	0.05948342	0.05735379
	0.1	0.05948342	0.05735379
θ	0.16	0.03736405	0.03650562
	0.2	0.02987800	0.02932280
	0.02	0.05948342	0.05735379
r	0.1	0.05332333	0.05166676
	0.2	0.04074747	0.04009666
	0.016	0.05948342	0.05735379
σ	0.02	0.05948343	0.05735380
	0.3	0.05948748	0.05736245

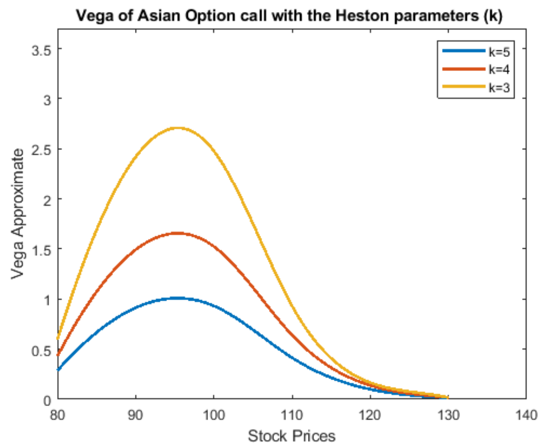

Figure 8: Vega of Asian Option with Heston Model parameters

Figure 8 shows that the Vega option is higher from OTM to ITM with a lower value of k , which is the mean reversion speed. This figure indicates that this pattern is more pronounced for ATM than ITM and OTM.

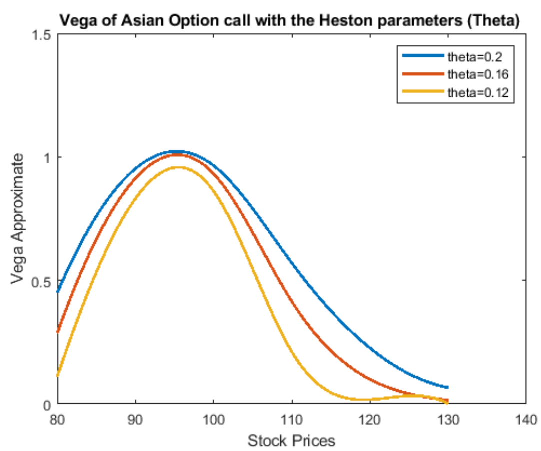

Figure 9: Vega of Asian Option with Heston Model parameters

Figure 9 shows that when θ is reduced then the values of Vega option for both from OTM to ITM are reduced. This pattern is more notable for the ITM and OTM options than the ATM options, as the three curves are almost intersecting at around where S is equal to the strike price K.

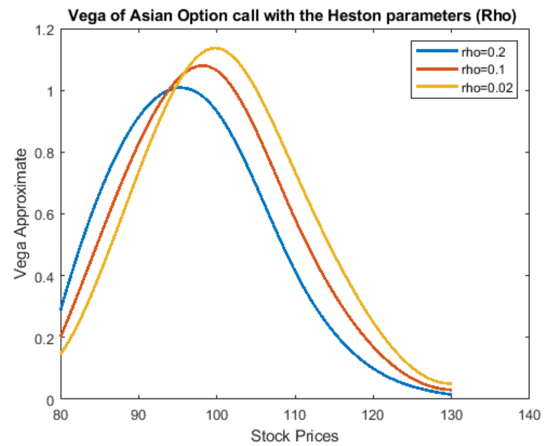

Figure 10: Vega of Asian Option with Heston Model parameters

Figure 10 shows that at around OTM, as the interest r grows, the value of Vega Approximate is higher and at around ITM, it shows us the opposite phenomena (the smaller the interest rate and the higher the value of approximate Vega). this pattern with r has the similar phenomena as the above with θ.

Furthermore, the figure 11 shows how Vanna oscillates with respect to the changes in the underlying asset S:

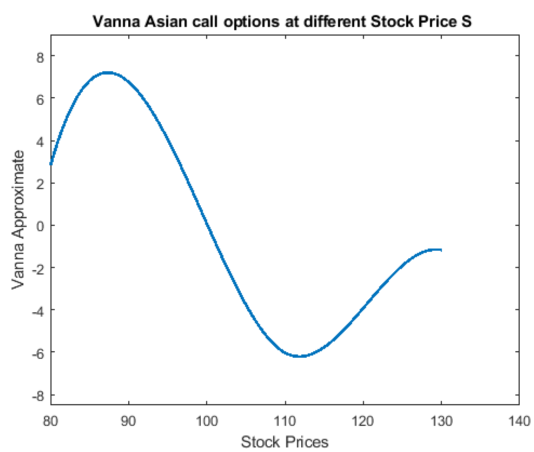

Figure 11: Vanna of Asian Option with Heston Model parameters

This figure shows that Vanna Approximate of Asian option has positive values when the underlying asset S is lesser than the strike K (OTM), and it has negative values when the underlying asset S is greater than the strike price K (ITM).

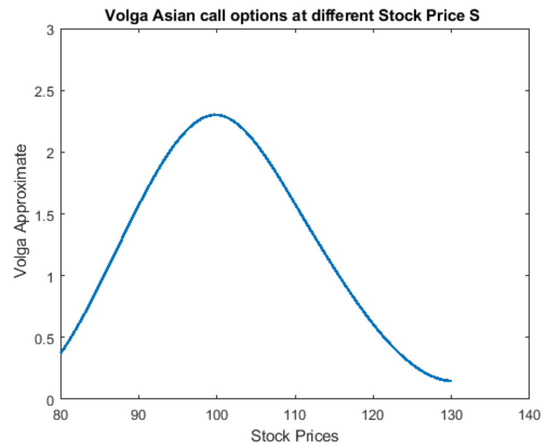

Figure 12: Volga of Asian Option with Heston Model parameters

Figure 12 depicts a similar pattern Figures 4.6, it is because for volatility movements, Volga Approximate is to Vega Approximate what Gamma Approximate is to Delta Approximate (for spot movement).

4.1 A Comparisons of some Greeks

Comparison is done by analyzing Sensitivities obtained from an approximation with the Heston Model Parameters and those from the Black-Scholes model see [17] and [23]. Consider an Asian option with the same scenario as previous section: $K = 100$; $T = 0.5$; $\sigma = 0.016$; $k = 5$; $\theta = 0.1$; $\rho = 0.01$; $r = 0.02$;and $v_0 = 0.0625$. For both figures 13 and 14, we consider the difference between the Greeks approximate with the Heston Model Parameters against those from [17] and [23]. Here, we only discuss about the Delta and the Gamma when the Stock Price (S) is from 60 to 130.

Figure 13 shows that earlier around OTM, Delta of Asian call option with the Heston Parameters is much higher than the Delta of Asian option obtained from [23] with the Black Schole Model, While for ITM options, Delta of Asian call option with the Heston Parameters is cheaper than Delta from Black Schole Model (see [23]). Furthermore, this figure shows the same pattern of Delta of Asian call option with Heston Model Parameters compare against Delta obtained from the large deviation theory (see [17]).

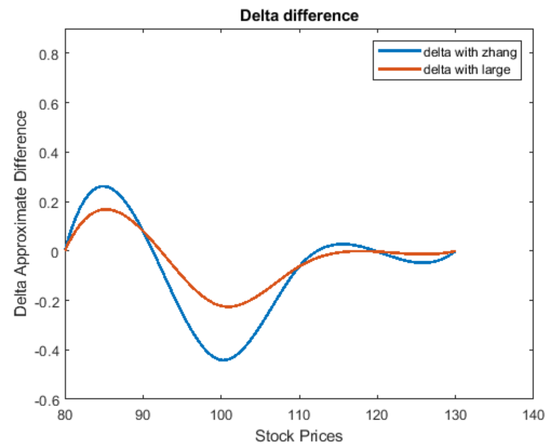

Figure 13: Delta Difference between Delta from Heston Model Parameters and Black-Scholes Model

Figure 14 shows that for the Gamma approximate with the Heston model Parameters is higher than Gamma obtained from [23] from OTM to ITM options. This figure shows also that Gamma approximate with the Heston model Parameters is higher than Gamma obtained from the Large deviation in [17] only around ATM options but for OTM and ITM we observe the opposite case. The option with higher Gamma have higher since an unfavorable move in the the underlying stock have an oversize impact.

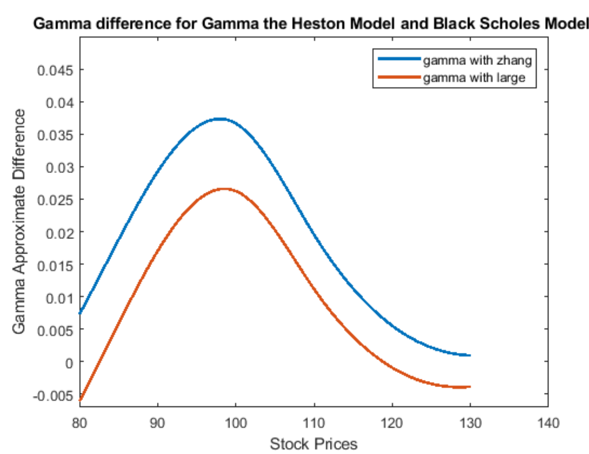

Figure 14: Gamma Difference between Delta from Heston Model Parameters and Black-Scholes Model

5 Conclusion

The valuation of Asian options is of both theoretical interest and practical importance. Several approaches have been proposed to value these options. This paper has provided an analytical formulas of Asian call options and theirs Greeks with Heston model parameters. A study of Greeks with different Heston model parameters shows the variations of the Greeks from out-of-the-money to In-the-money options. Finally, a comparison of these approximations against those from [17] and [23], confirm the better performance of the Heston model against the Black-Scholes model.

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References

[1] L.-P. Arguin, N.-L. Liu and T.-H. Wang, Most-likely-path in asian option pricing under local volatility models, *International Journal of Theoretical and Applied Finance*, **21**, (2018), 1850029.

[2] F. Black and M. Scholes, The pricing of options and corporate liabilities, *Journal of political economy*, **81**, (1973), 637–654.

[3] P. Boyle, M. Broadie and P. Glasserman, Monte carlo methods for security pricing, *Journal of economic dynamics and control*, **21**, (1997), 1267–1321.

[4] M. Broadie and P. Glasserman, Estimating security price derivatives using simulation, *Management science*, **42**, (1996), 269–285.

[5] M. Broadie and O. Kaya, Exact simulation of option greeks under stochastic volatility and jump diffusion models, in: Simulation Conference, 2004. Proceedings of the 2004 Winter, IEEE, **2**, 1607–1615.

[6] J. Choi and K. Kim, The derivatives of asian call option prices, *arXiv preprint arXiv:0712.1093*, (2007).

[7] M. Curran, Valuing asian and portfolio options by conditioning on the geometric mean price, *Management science*, **40**, (1994), 1705–1711.

[8] E. Fournié, J.-M. Lasry, J. Lebuchoux, P.-L. Lions, N. Touzi, Applications of malliavin calculus to monte carlo methods in finance, *Finance and Stochastics*, **3**, (1999), 391–412.

[9] J.-P. Fouque, C.-H. Han, et al., Pricing asian options with stochastic volatility, *Quantitative Finance*, **3**, (2003), 353–362.

[10] M.C. Fu, D.B. Madan and T. Wang, Pricing continuous asian options: a comparison of monte carlo and laplace transform inversion methods, *Journal of Computational Finance*, **2** (1999), 49–74.

[11] P. Glasserman, *Monte Carlo methods in financial engineering*, v.**53**, Springer Science & Business Media, 2003.

[12] S.L. Heston, A closed-form solution for options with stochastic volatility with applications to bond and currency options, *The review of financial studies*, **6**, (1993), 327–343.

[13] J. Hull and A. White, The pricing of options on assets with stochastic volatilities, *The Journal of Finance*, **42**, (1987), 281–300.

[14] A.G. Kemna and A.C. Vorst, A pricing method for options based on average asset values, *Journal of Banking & Finance*, **14**, (1990), 113–129.

[15] A.L. Lewis, *Option Valuation Under Stochastic Volatility Ii*, Finance Press, Newport Beach, CA, 2009.

[16] R.C. Merton, Theory of rational option pricing, *The Bell Journal of economics and management science*, (1973), 141–183.

[17] D. Pirjol and L. Zhu, Sensitivities of asian options in the black–scholes model, *International Journal of Theoretical and Applied Finance*, **21**, (2018), 1850008.

[18] D. Pirjol and L. Zhu, Short maturity asian options for the cev model, *Probability in the Engineering and Informational Sciences*, (2018), 1–33.

[19] O. Reiß and U. Wystup, *Efficient computation of option price sensitivities using homogeneity and other tricks*, WIAS, 2000.

[20] L.C.G. Rogers and Z. Shi, The value of an asian option, *Journal of Applied Probability*, **32**, (1995), 1077–1088.

[21] L.O. Scott, Option pricing when the variance changes randomly: Theory, estimation, and an application, *Journal of Financial and Quantitative analysis*, **22**, (1987), 419–438.

[22] S.M. Turnbull and L.M. Wakeman, A quick algorithm for pricing european average options, *Journal of financial and quantitative analysis*, **26**, (1991), 377–389.

[23] J. Zhang, A semi-analytical method for pricing and hedging continuously sampled arithmetic average rate options, *Journal of Computational Finance*, **5**, (2001), 59–80.

[24] J. Zhu, *Applications of Fourier transform to smile modeling: Theory and implementation*, Springer Science & Business Media, 2009.