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Impacts of a regime switching (elections) in investment

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Abstract

In this paper, we assess the impact on the timing of investment and option value for green energy company after election. For this fact, many parameters were taken into consideration such as the discount rate, the investment cost,the mean reversion speed, the mean reversion level, the uncertainty parameter, etc. The distribution of ω (advantages on the biological fuel due to the taxes system that can tolerate the use of it) after election is not known and is assumed to be uniformly distributed(Laplace′s insufficient reason criterion). The simulation study was conducted and the optimal investment time together with the option value for the company were found considering the subsidies and lower taxes that can be made depending on regime switching.

Keywords: Investment; Real options; Uniform distribution; Prices; Ambiguity

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1 Introduction

Elections are very important in every country, this represents a democracy by linking the interests of voters to those in government and allowing citizens to select representatives who reflect their opinions. The politicians or political parties try to conquer voting with the use of laws.
With a government elected by its citizens and that effects every aspect of their lives from schools to health care including investment to homeland security, voting is an important right in every society. By voting, you are making your voice heard and registering your opinion on how you think the government should operate. Political discussions on whether, when, and how to support technology projects can be a powerful deterrent to immediate investments because it creates an incentive to wait until a policy decision is made.

Elected government can fix a price advantage of biologic fuel used in green energy investment to promote those who are investing in the later given the importance of it for the country.

The price advantage of biologic fuel is mainly driven by political decisions, it can be expected that changing political majorities may have an impact on its size. Some political parties might claim to reduce the price advantage of biologic fuel and other political parties might claim to increase this advantage or not having this advantage at all. Then, the exact amount of this advantage will depend on public opinion, future election results and the results of coalition negotiations[4].

As Knight saw it, an ever-changing world brings new opportunities for businesses to make profits, but also means we have imperfect knowledge of future events. Therefore, according to Knight, risk applies to situations where we do not know the outcome of a given situation, but can accurately measure the odds. Uncertainty, on the other hand, applies to situations where we cannot know all the information we need in order to set accurate odds in the first place. "There is a fundamental distinction between the reward for taking a known risk and that for assuming a risk whose value itself is not known," said Knight. A known risk is easily converted into an effective certainty, while true uncertainty, as Knight called it, is not susceptible to measurement. An airline might forecast that the risk of an accident involving one of its planes is exactly one per 20 million takeoffs. But the economic outlook for airlines 30 years from now involves so many unknown factors as to be incalculable.

Therefore, in contrast to the economic price uncertainty the political uncertainty is an example of Knightian uncertainty or ambiguity, i.e. its probability distribution is unknown[3],[5],[6].

The remainder of the paper is structured as follows: in Section 2, we present the price model; in Sections 3, we include in the model the political ambiguity, then in section 4, we present the distribution of the advantage that can be done to the price of the fuels. Section 5 deals with the simulation approach used to estimate the investment timing and option value. We offer concluding remarks in Section 6.

2 Model

We consider a risk-neutral company that discount with a risk-free interest rate $r > 0$. The company own an asset that at time $t_0 = 0$ have the finite life-time of $\tau > 0$ and that consume x units of fuel per time unit. The price $p(t)$ of fuel evolves stochastically over time and follows the Ornstein-Uhlenbeck process

$$dp(t) = k\,(m - p(t))\,dt + \sigma dW_t, \quad p(0) = p_0 \geq 0 \tag{1}$$

where $k > 0$ is the mean reversion speed which is the assumption that a price of stock will tend to move to the average price over time, m is the mean reversion level, $\sigma > 0$ is the uncertainty parameter and dW_t is the increment of Weiner process with a normal distribution [1],[7].

During the life-time of the asset the company has at any time the opportunity (option) to adjust its asset in a way that it can also tolerate biologic fuel. Mainly driven by subsidies and lower taxes the price $p_B(t)$ of biologic fuel is cheaper than the price of standard fuel and depends on the time for which the option is bought. In particular, we assume that

$$p_B(t) = (1 - \xi)\,p(t)$$

where $0 \leq \xi < 1$.

From (1) we can derive

$$p(t) = e^{-kt}p_0 + \left(1 - e^{-kt}\right)m + \sigma \int_0^t e^{ks} dW(s)$$

and

$$\mathbb{E}\left(p(t_2)|p(t_1)=p_{t_1}\right)=e^{-k(t_2-t_1)}p_{t_1}+\left(1-e^{-k(t_2-t_1)}\right)m+\frac{\sigma^2\left(1-e^{-k(t_2-t_1)}\right)}{k}$$

for every $t_2 \geq t_1 \geq 0$.

If the company invests at a time $t > 0$, therefore, it expects discounted savings of

$$V\left(p(t),t\right)=\mathbb{E}\int_t^\tau x\left(p(s)-p_B(s)\right)e^{-r(s-t)}ds=x\xi\int_t^\tau \mathbb{E}\left(p(s)|p(t)\right)e^{-r(s-t)}ds$$

Thus, investing at time t generates an expected profit of

$$\pi\left(p(t),t\right)=V\left(p(t),t\right)-I(t)$$

where $I(t)$ is the investment costs.

The cashflow process Y evolves as follows:

$$dY(t)=Y(t)\left(\alpha dt+\zeta\left(\rho dW(t)+\sqrt{1-\rho^2}dW^0(t)\right)\right) \tag{2}$$

where $W^0(t)$ is a Wiener process, $\rho^2 < 1$ is the correlation coefficient between market uncertainty and the cashflow process uncertainty, and α, ζ are all constants.

Following [2], the possibility to invest can be regarded as a real option. Hence, the company should not invest immediately but wait with the investment until the price of standard fuel reaches the time-depending optimal threshold $p^*(t)$ [1].

3 Political ambiguity in the model

Let us now, for the next, integrate this political uncertainty into the model and assume that this price advantage may switch at a known point of time $T > 0$ (election day: Te) and that the price of biologic fuel equals

$$p_B(t)=\begin{cases}(1-\xi)\,p(t);t_0\leq t<T\\ \omega\,(1-\xi)\,p(t);T\leq t\leq\tau\end{cases} \tag{3}$$

Where ξ indicates the result of the price advantage of biologic fuel for the elected government. The higher is this result the lower is the price advantage of biologic fuel after the election date and the lower is this result the higher is the price advantage of biologic fuel after the election date. If $\omega = 0$, the biologic fuel will be free. If $\omega = 1$, the price of biologic fuel before and after election will not change and if $\omega = \frac{1}{1-\xi}$, there will be no advantage at all for the price of biologic fuel [8].

Then the savings can be given by

$$x\left(p(t) - p_B(t)\right) = \begin{cases} x\xi p(t); t_0 \leq t < T \\ x\left(1 - \omega + \omega\xi\right) p(t); T \leq t \leq \tau \end{cases}$$

If the company invests at time $t > 0$ it will expect a discounted savings of

$$\begin{aligned} V_\omega\left(p(t), t\right) &= \mathbb{E} \int_t^\tau x\left(p(s) - p_B(s)\right) e^{-r(s-t)} ds \\ &= \mathbb{E} \int_t^T x\xi p(s) e^{-r(s-t)} ds + \mathbb{E} \int_T^\tau x\left(1 - \omega + \omega\xi\right) p(s) e^{-r(s-t)} ds \\ &= x\xi \int_t^T \mathbb{E}\left(p(s)|p(t)\right) e^{-r(s-t)} ds + x\left(1 - \omega + \omega\xi\right) \int_0^\infty \Psi_p\left(y, p(t), \tau, T\right) \\ &\int_T^\tau \mathbb{E}\left(p(s)|y\right) e^{-r(s-t)} dsdy \end{aligned}$$

where $\Psi_p\left(y, p_T, \tau, T\right) := \dfrac{\partial \mathbb{P}\left(p(\tau) \leq y | p(T) = p_T\right)}{\partial y}$ denotes the transition density function of the price process p which can be obtained via a Monte-Carlo simulation.

For $\tau \longrightarrow \infty$, the Ornstein-Uhlenbeck process approaches a stationary Gaussian process with zero mean, called the stationary Ornstein-Uhlenbeck process. Unlike Brownian paths, whose fluctatations grow with time, the stationary Ornstein-Uhlenbeck paths consist of fluctuations that are typically of the order σ, although larger fluctuations occur over long enough times.

Thus, investing at time $t > 0$ generates an expected profit of

$$\pi_\omega\left(p(t), t\right) = V_\omega\left(p(t), t\right) - I(t)$$

where $I(t)$ is the investment costs.
Not investing until T will generate an option value at time T of

$$F(p(T), T) = \int_a^b \frac{1}{b-a} F_z(p(T), T) dz$$

where $F_z(x) := \dfrac{\partial \mathbb{P}(z \leq x)}{\partial z}$
Or equivalently

$$F(p(T), T) = \max\{S_t - I, e^{-rdt} \mathbb{E}^P\left[dV_t | F_t\right]\}$$

The value $F_\omega(p(t),t)$ of the option to invest is the solution of the differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F_\omega(p,t)}{\partial^2 p} + k(m-p)\frac{\partial F_\omega(p,t)}{\partial p} + \frac{\partial F_\omega(p,t)}{\partial t} = rF_\omega(p,t)$$

and also meets the following conditions:

1. Zero is an absorbing barrier of the price process, i.e
$\lim_{p(t)\to 0} F_\omega(p(t),t) = 0$, $\forall t \geq 0$

2. The investment opportunity has no value if the asset is no longer in use,
$F_\omega(p(t),t) = 0$, $\forall t \geq \tau$

3. Continuity-condition, i.e.
$F_\omega(p^*_\omega(t),t) = \Pi_\omega(p^*_\omega(t),t)$

4. Smooth-pasting condition, i.e
$\dfrac{\partial F_\omega(p^*_\omega(t),t)}{\partial p} = \dfrac{\partial \Pi_\omega(p^*_\omega(t),t)}{\partial p}$

The optimal investment time for the company is determined by

$$t^*_\omega = inf\{t \geq t_0 : p(t) \geq p^*_\omega(t)\}$$

where $p^*_\omega(t)$ is the point representing the optimal threshold [1].

4 Distribution of ω

The company does not know the exact value of ω before the election. We can consider its distribution like uniform distribution on $[\omega_{min}, \omega_{max}] =: [a,b]$ from the election date T to the stoping time τ (the Laplace′s insufficient reason criterion postulates that if no information is available about the probabilities of the various outcomes, it is reasonable to assume that they are likely equally)[9]. Its probability density function is

$$f_\omega(z) = \begin{cases} \frac{1}{b-a}; a \leq z \leq b \\ 0; elsewhere \end{cases}$$

Its expected value and variance are respectively given by $\frac{a+b}{2}$ and $\frac{1}{12}(b-a)^2$ and its cumulative distribution function is

$$F_\omega(z) = \begin{cases} 0, z < a \\ \frac{z-a}{b-a}; a \leq z \leq b \\ 1; z > b \end{cases}$$

Here a represents the highest demanded promotion of biologic fuel and b the lowest demanded promotion of biologic fuel.

Therefore, if there are n outcomes, the probability of each outcome will be $\dfrac{1}{n}$ since the outcomes are likely equal. When we calculate the payoff for each alternative, we consider the alternative with the largest value of the payoff.

For each state of nature ($S_j, j = 1, \cdots, n$ in S), the decision maker should assign the probability of $p_j = \dfrac{1}{n}$ that S_j will occur.
For each decision alternative (controllable variable in the system) $A_i, i = 1, \cdots, m$, we will compute $\mathbb{E}(A_i) = \sum_{j=1}^{n} p_j R_{ij}$, where R_{ij} are the payoffs (rewards) obtained by choosing alternative A_i if state S_j (uncertain event or state of nature) occurs. The action alternative with the best $\mathbb{E}(A_i)$ is the one corresponding to the optimal decision [10].

Assuming the uniform distribution; the company expects,if it invests at time T, discounted savings of (by calculating the expected payoff for each alternative and select the alternative with the largest value) [12]

$$\begin{aligned} V\left(p(t), t\right) = {} & x\xi \int_t^\tau \frac{\mathbb{E}\left(p(s)|p(t)\right)}{e^{r(s-t)}} ds + x\left(\xi - 1\right) \int_a^b \frac{z-1}{b-a} \int_0^\infty \Psi_p\left(y, p(t), \tau, T\right) \\ & \int_T^\tau \frac{\mathbb{E}\left(p(s)|y\right)}{e^{r(s-t)}} dsdydz \\ = {} & x\xi \int_t^\tau \frac{\mathbb{E}\left(p(s)|p(t)\right)}{e^{r(s-t)}} ds + x\left(\xi - 1\right) \left(\frac{b+a}{2} - 1\right) \int_0^\infty \Psi_p\left(y, p(t), \tau, T\right) \\ & \int_T^\tau \frac{\mathbb{E}\left(p(s)|y\right)}{e^{r(s-t)}} dsdy \end{aligned}$$

Apart from the equal distribution, we can also be interested in evaluation of the investment using the optimism.
For this we consider a as the highest demanded promotion of biologic fuel and b the lowest demanded promotion of biologic fuel. If the the actual value of the promotion after the election date is ω, it is clear that $a \leqslant \omega \leqslant b$ and $0 \leqslant a \leqslant b \leqslant \frac{1}{1-\xi}$.

The pessimistic decision-maker thinks that after election b will be applied and the optimistic decision-maker thinks that after election a will be applied. For both of them, the expected profit will be $\pi_b(p(t),t)$ and $\pi_a(p(t),t)$ respectively; the option to invest has a value of $F_b(p(t),t)$ and $F_a(p(t),t)$ respectively; and finally they invest respectively at time

$$t_b^* = inf\{t \geq t_0 : p(t) \geq p_b^*(t)\}$$

and

$$t_a^* = inf\{t \geq t_0 : p(t) \geq p_a^*(t)\}$$

The pessimistic decision-maker decides according to the Maximin decision rule as described by [11] and the optimistic decision maker decides according to the Maximax decision rule. These two rules were combined by Hurwicz (1951) by using the optimism parameter $0 \leqslant \lambda \leqslant 1$.
For this, the decision-maker thinks that investing at time $t_0 \leqslant t \leqslant T$ would generate an expected profit of

$$\pi_\lambda(p(t),t) = \lambda\pi_b(p(t),t) + (1-\lambda)\pi_a(p(t),t)$$

And he assumes that not investing until T will generate an option value at time T of

$$F_\lambda(p(T),T) = \lambda F_b(p(T),T) + (1-\lambda)F_a(p(T),T)$$

The value $F_\lambda(p(t),t)$ of the option to invest as well as the optimal investment threshold $p_\lambda^*(t)$ are obtained by solving the following differential equation

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F_\lambda(p,t)}{\partial^2 p} + k(m-p)\frac{\partial F_\lambda(p,t)}{\partial p} + \frac{\partial F_\lambda(p,t)}{\partial t} = rF_\lambda(p,t)$$

and also meets the following conditions:

1. $\lim_{p(t)\to 0} F_\lambda(p(t),t) = 0, \forall t \geq 0$

2. $F_\lambda(p(T),T) = \lambda F_b(p(T),T) + (1-\lambda)F_a(p(T),T)$

3. $F_\lambda(p_\lambda^*(t),t) = \Pi_\lambda(p_\lambda^*(t),t)$

4. $\dfrac{\partial F_\lambda(p_\lambda^*(t),t)}{\partial p} = \dfrac{\partial \Pi_\lambda(p_\lambda^*(t),t)}{\partial p}$

The optimal investment time for the company is determined by[13]

$$t^*_\lambda = inf\{T \geq t \geq t_0 : p(t) \geq p^*_\lambda(t)\}$$

where $p^*_\lambda(t)$ is the point representing the optimal threshold.

5 Simulation Study

The following parameters were used: $a = 0.75$; $b = 1.25$; $k = 0.5$; $p_0 = 1.5$; $m = ln(1.5)$; $\sigma = 0.2$; $\tau = 10$;$\lambda = 0.5$; $\eta = 0.5$; $\xi = 0.3$; $I_f = 175$; $I_v = 100$;$T = 3$ and $x = 100$.

Table 1 indicates the similated values of the advantages that can be applied after election (ω). The last has the uniform distribution on $[0.75, 1.25]$ from the election date $T = 3$ to the stoping time $\tau = 10$.

Considering the advantages and taking the mean of ω, using (3),one can find the prices of biological fuel which indicate the different advantages (promotion) the new government can implent after elections (see Table 2). The different advantages before election are also shown in tTable 2, and it is clear that there is a good promotion before election than after election.

```
 [1] 0.8289831 1.2378960 1.0011211 1.0536575 1.0095414 0.9176651 1.0234325
 [8] 0.8573524 0.9029793 0.7693657 1.0009064 0.9957780 1.1070278 1.0098026
[15] 1.1359956 1.1963146 0.9658213 0.8862352 0.9266843 1.1527945 0.8280408
[22] 1.2413777 1.1887878 0.7628744 0.9193730 0.8010972 1.0678175 1.0228310
[29] 1.2290344 1.1348722 0.8344454 0.9018525 0.7641937 0.7711437 1.1157866
[36] 1.0395556 0.7967844 0.8015701 1.0802797 0.8972139 0.8384093 1.2482338
[43] 0.8998700 1.2016326 1.2184388 0.9042776 1.1590995 0.8025506 1.0091345
[50] 1.1862663 0.9572116 0.7508529 1.0992830 0.9564951 1.0487682 1.1884807
[57] 1.1032238 1.0289427 1.2181914 0.7986240 0.9913652 0.8753749 1.0342056
[64] 0.7902533 0.7562870 0.9843676 0.8478655 1.0694964 1.1969993 1.1541587
[71] 1.0273818 1.2267753 0.9277247 0.8919714 1.0592516 1.0869542 1.1098552
[78] 1.1315990 1.0922666 1.2068212 0.8355534 0.7653346 1.1301204 0.9578221
[85] 0.9301274 0.8977373 1.1888965 1.1607987 0.8935811 0.8531169 0.8427432
[92] 0.9197642 0.8205270 1.0471091 0.9064381 1.1536658 1.0936542 0.8197430
[99] 1.0810562 1.0612290
```

Table 1: Structure of BP network

	t	Advantages
1	0	1.7665
2	0.5	1.7634
3	1	1.7204
4	1.5	1.8382
5	2	1.9694
6	2.5	2.0816
7	3	2.38
8	3.5	2.5182
9	4	2.9385
10	4.5	3.2717
11	5	3.4229
12	5.5	4.3761
13	6	5.6304
14	6.5	6.8202
15	7	6.9031
16	7.5	8.0307
17	8	14.0161
18	8.5	15.3609
19	9	15.5841
20	9.5	18.5026
21	10	25.8249

Table 2: Advantages on fuel

Using those parameters, we have got that the optimal investment time for the company which is determined by

$$t_\omega^* = inf\{t \geq t_0 : p(t) \geq p_\omega^*(t)\}$$

is obtained after 0.5 year before election as it can be seen on Figure 1 and at that time the coresponding option value is 214.178 (see Figure 2).
The same procedure was applied after 3 years (election time) and we found that the optimal investment time for the company is obtained after 4 years, or 1 year after election as it can be seen on Figure 1 and at that time the coresponding option value is 35.513 (see Figure 2).

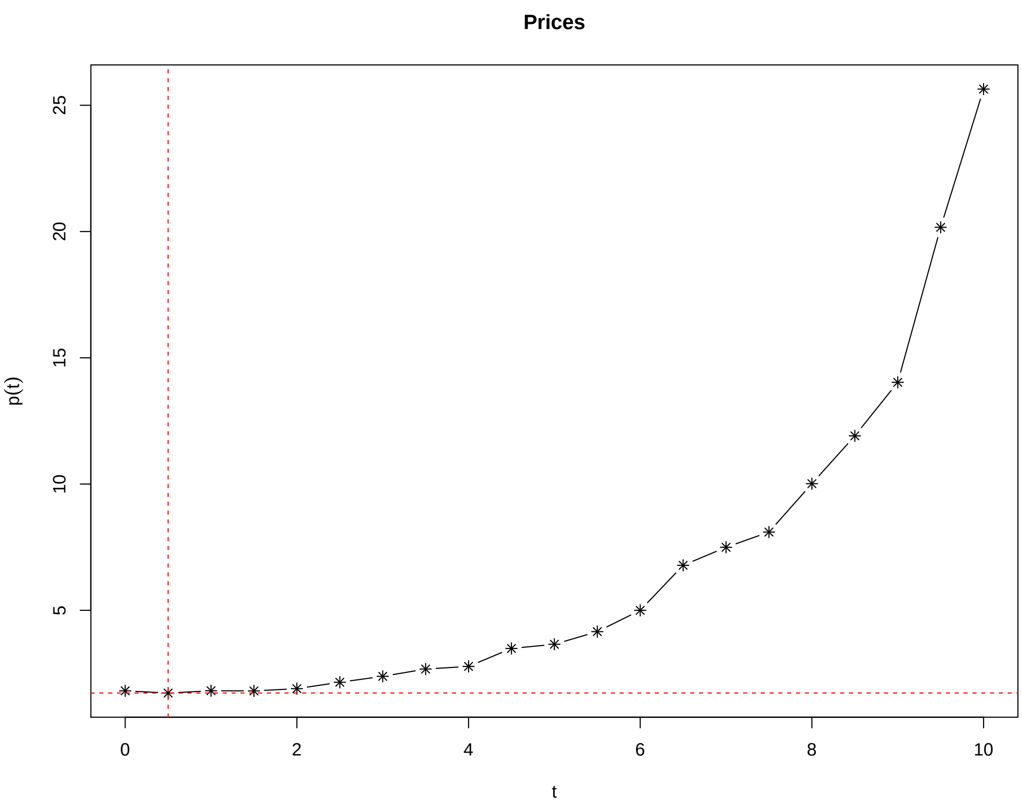

Figure 1: Plot of the similated prices

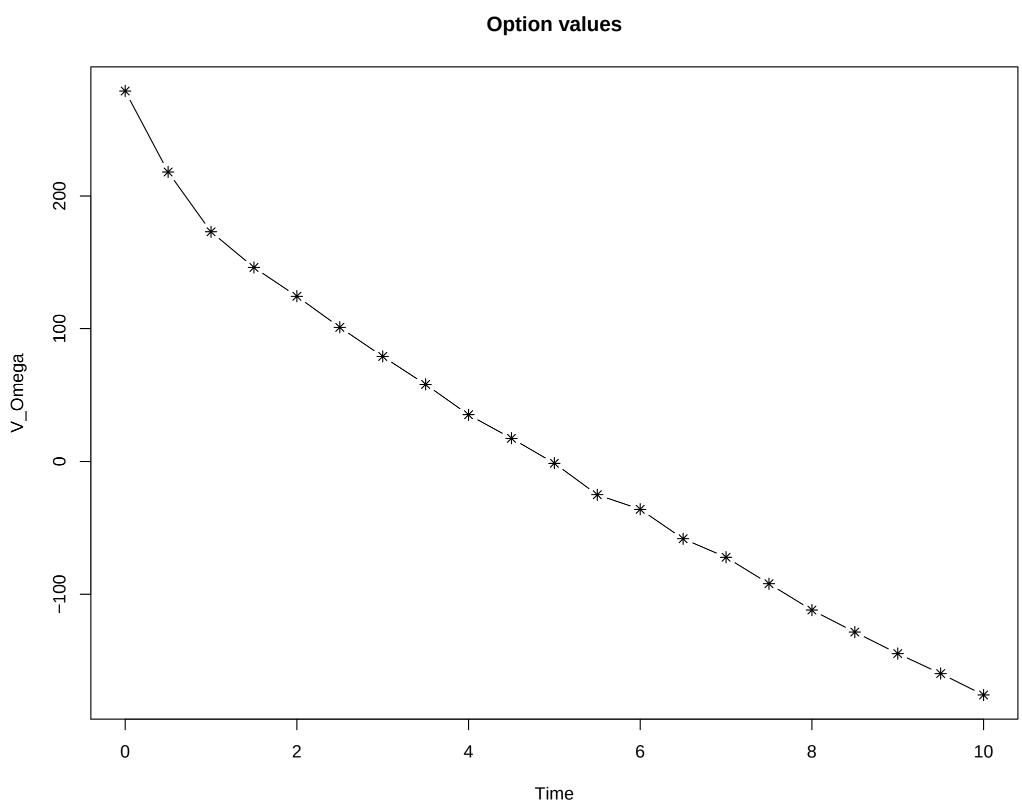

Figure 2: Plot of the similated option values

6 Conclusion

Green energy is very important everywhere. For this study, the price of fuel $p(t)$ evolves stochastically and follows the Ornstein-Uhlenbeck process. The regime switching, after 3 years may cause some changes on the advantages given to the biological fuel that the company can also use in the production of the green energy. The advantage is decreasing with the time and the political ambiguity is greater the shorter the remaining time to the election. Table 2 gives different values which represent the results on the advantages on the biological fuel, from this table as from (3) the higher is this result the lower is the price advantage of biologic fuel after the election date and the lower is this result the higher is the price advantage of biologic fuel after the election date, and this will affect the savings of the company given that the price $p(t)$ is increasing.

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