# Kautilya, Fibonacci and Samuelson on Discounting 

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#### Abstract

The concept of discounting plays an important role in allowing efficient inter-temporal choices. William Goetzmann (2004) claims that Fibonacci (1202/2002) introduced discounting. (i) It is shown that Fibonacci did not understand the concept of discounting or its relevance and his Liber Abaci is a book only of calculations and not of concepts. (ii) It is claimed that Kautilya ( $4^{\text {th }}$ century BCE) originated the concept of discounting and inter-temporal analysis. (iii) More importantly, relevance of his insight to today's world is brought out. According to Kautilya, it was possible to reduce the risk premium by making rare disasters rarer. Samuelson (1964) made the same point but Jensen and Bailey (1972) completely missed it.


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## 1 Introduction

Fisher (1931) acknowledged the contributions of John Rae and of Eugen von Böhm-Bawerk. Fredrick, Shane, George Loewenstein and Ted O’Donoghue (2002) credit John Rae (1834) for introducing inter-temporal analysis. Apparently, William Goetzmann (2004) is the only one, who claims that Fibonacci (1202/2002) introduced discounting. Actually, Kautilya, more than one thousand years before Fibonacci and two thousand years earlier than Rae, introduced the concept of discounting. He was concerned not just with the time dimension of an investment

[^0]but also with its return and risk. Section 2 presents Kautilya's ideas on discounting. There are essentially two examples in Chapter 12 of Liber Abaci that form the basis of Goetzmann's conclusion. It is shown that Fibonacci understood neither the concept of discounting nor its significance and his Liber Abaci, as he intended, is a book only of calculations and not of concepts. This is discussed in Section 3. Kautilya argued that projects that enhanced national security would reduce the probability of political instability and that would lower the risk premium. Samuelson (1964), perhaps, is the only other economist who suggested giving preference to projects that enhance national security. It is pointed out that Jensen and Bailey (1972) misunderstand Samuelson's core argument. This is discussed in Section 4 and Section 5 contains conclusion.

## 2 Kautilya on Discounting

"A pigeon today is better than a peacock tomorrow." Kautilya (4th Century BCE/ 2000)
One may wonder why anyone would need the concept of discounting during the fourth century BCE. Kautilya believed that poverty was death while living. He wrote The Arthashastra, a manual on engineering shared prosperity (see Sihag (2014, 2016) for an in-depth analysis). It contains a conceptual framework along with operational details to promote Yogakshema-peaceful enjoyment of prosperity-of every citizen. He introduced not just the concept of discounting but more than a score of other key concepts to operationalize his vision of lifting people out of poverty. ${ }^{2}$
Kautilya understood that selection of most productive projects was essential to bringing prosperity. He realized that selection or evaluation of projects was not possible without the availability of accounting methods. He developed reasonably

[^1]Table 1: Concepts and Ideas Developed and Used by Kautilya

| Re-emerged <br> during the <br> period | Concepts and Ideas Originated and applied by Kautilya |
| :--- | :--- |
| $1700-1850$ | Gains from trade, diversification, Division of labor, Inter-temporal choice, labor theory of property, Law <br> of diminishing returns, moral hazard, regulation of monopoly, sources of economic growth, principles of <br> taxation |
| $1850-1900$ | Distinction between short-run and long run, Efficiency Wages, externality, Duipit-Laffer Curve <br> (Kautilya Curve), Demand-Supply Apparatus, Opportunity cost, Producer Surplus |
| $1900-1970$ | Principal-agent problem, Liquidity, Mean-Variance approach, non-cooperative game |
| $1970-$ Present | Asymmetric information, piece-wise Linear income Tax, Loss-aversion, information economics, <br> Self-protection, self-insurance, Time Inconsistency, Systemic risk |

advanced accounting methods. ${ }^{3}$ Then, he proceeded to identify the concepts of discounting and opportunity cost which were also considered essential to the selection of projects. ${ }^{4}$
There are at least two paragraphs in The Arthashastra where inter-temporal arbitrage is discernible. (i) Kautilya (p 636) suggested, "When the gains from two campaigns are equal, the king shall compare the following qualities and choose the one which has more good points: place and time; the power and the means required to acquire it; the pleasure or displeasure caused by it; speed or slowness of getting it; the proximity or distance, the immediate and future consequences; its high value or constant worth; and its abundance or variety (9.4)." Discounting: Although Kautilya did not make the Fisherian distinction between impatience and investment opportunity, but he did understand the concept of discounting and suggested its application. However, he concentrated only on the investment opportunity side since he needed it for the selection or evaluation of projects.
Kautilya (p 635) wrote, "A great gain is a substantial gain available immediately (9.4)." However, if the gains from the two campaigns were spread over time, selection should depend on 'speed or slowness of getting it'. As an illustration suppose the gains from two campaigns were equal, say $\$ 1500$. The gains from one campaign accrue at a rate of $\$ 500, \$ 500$ and $\$ 500$ over three periods and the gains from the second campaign accrue at the rate of $\$ 300, \$ 300, \$ 300, \$ 300$ and $\$ 300$ over five periods. According to Kautilya, the king should prefer the first one. It is obvious that Kautilya introduced the concept of discounting and initiated the discussion on making inter-temporal choices.
Importance of Place: He explained, in addition to the time dimension, the other

[^2]attributes as to why they were relevant to the decision. For example, he preferred a safer location of the piece of land. He (p 618) asked, "Which is better--a rich land with permanent enemies or poor land without permanent enemies?" He answered it as follows: "Some teachers say that, because a rich land enables one to get wealth and an army with which to destroy the enemies, a rich land with permanent enemies is preferable. Kautilya disagrees. Acquiring land with such enemies, one only adds to one's number of enemies; and an enemy remains an enemy whether he is helped or harmed: on the other hand, a temporary enemy can be made to be quiet through favours or at least by not harming him (7.10)." Again, he preferred an irrigated piece of land to a dry piece of land. He (p 619) stated, "As between land dependent on rain and land with flowing water [i.e. a river], a smaller tract with flowing water is preferable to a larger drier one because with flowing water, which is always available, the production of crops is assured. As between two irrigated tracts, one on which cereals can be grown is preferable. (7.11)."

He was clearly aware of the risk-return trade-off. According to him, if the gains from two campaigns were the same, the one, which had a stable worth, and was available quickly was preferable. Incidentally, the phrase 'the immediate and future consequences' indicates that he made a distinction between short-run and long run.
(ii) Another example of Inter-Temporal Analysis: He explicitly incorporates both risk and return in making a choice. He (p 617) asked, "Which is preferable-an immediate small gain or a large gain in the future?" According to him, the answer depended on two factors: "A large gain in the future is preferable if it is like a seed [yielding fruit in the future] and if it is not likely to disappear [before fruition]. Otherwise, [if there is no growth and if there is a danger of it not fructifying] the small immediate gain is preferable (7.9)." He explained that the choice between accepting a small gain immediately and a large gain in the future depended on two factors. First, what was the nature of the gain? Was it like a seed, which had a potential for growth? Secondly, how certain it was? Was it likely to disappear before it materialized?

## 3 Fibonacci on Discounting

Liber Abaci was written in 1202 and for the next 800 years no one found the concept of discounting in it. Fisher dedicated his Theory of Interest to "the memory of John Rae and of Eugen von Böhm-Bawerk, who laid the foundations upon which I have endeavored to build." There are two problems and their solutions in Chapter 12 of Liber Abaci, which give the false impression that Fibonacci understood the concept of discounting. However, based essentially on Fibonacci's solutions to these two, specifically the second one, problems, Goetzmann credits him for introducing the concepts of discounting and present value. He claims, "This problem alone is unmatched in the history of financial
analysis. Although the mathematics of interest rates had a 3,000 year history before Fibonacci, his remarkable exposition and development of multi-period discounting is a quantum leap above his predecessors". It is shown that this is really a case of 'irrational exuberance' (an exaggerated claim).
Problem 1: Fibonacci (1202, p 372) asks, "A certain man proceeding to Lucca on business to make a profit doubled his money, and he spent 12 denari. He then left and went through Florence; but there doubled his money; and he spent 12 denari. Then he returned to Pisa, doubled his money, and spent 12 denari, and it is proposed that he had nothing left. It is sought how much he had at the beginning." Fibonacci's Solution: Fibonacci provides two methods to solve the above problem. ${ }^{5}$ He (p 372) describes the first method as: "Because it is proposed that he always doubled his money, it is clear that 2 will be made from one. Whence it is seen what fraction 1 is of 2 , namely $1 / 2$, which thus is written three times because of the three trips he made: $(1 / 2)(1 / 2)(1 / 2)$, and the 2 is multiplied by the 2 and the other twos that are under the fraction; there will be 8 of which you take $1 / 2$, namely 4 , of which you take $1 / 2$, namely 2 , and of the 2 you take $1 / 2$, namely 1 . After this you add the 4 to the 2 and the 1 ; there will be 7 that you multiply by the 12 denari which he spent; there will be 84 that you divide by the 8 ; the quotient will be $(1 / 2) 10$, and the man had this money."
Laurence Sigler describes Fibonacci's methodology for solving the problems. He (pp 8-9) writes, "The method of single false position solves which are equivalent to linear equations of the simple type $\mathrm{Ax}=\mathrm{B}$, and double false position which is used on problems leading to equations of the type $A x+B=C$. Double false position problems are found in chapter 13. Besides the method of false position Leonardo solves problems using what he calls the direct method. This method involves calling the sought quantity the thing and creating an equation containing the thing. The equation is stated in sentences and not written symbolically as we do today. The equation is then solved step by step for the thing. This is, of course, algebra as we know it, and is precisely the method described by al-Khwarizimi in his book on algebra."

[^3]According to Laurence Sigler, Fibonacci chose to use sentences instead of algebra to find solutions to the problems. He notes, "This is, of course, algebra as we know it, and precisely the method described by al-Khwarizimi in his book on algebra." In all probability, Fibonacci had an equation in the background. It does not require any stretch of imagination that he used the following method to find the answer. Suppose the merchant starts with X number of denari. His money doubles at Luca, that is, he has 2 X . He spends 12 denari. He is left with $2 \mathrm{X}-12$. His money again doubles, that is he has $2(2 \mathrm{X}-12)$. He spends 12 denari, he is left with $(2(2 \mathrm{X}-12)-12)$. He again doubles his money, that is he has $2(2 \mathrm{X}$ $-12)-12$ ). He spends 12 denari and he is not left with any money. That means:

$$
\begin{aligned}
& 2(2(2 \mathrm{X}-12)-12)-12=0 \\
& 2 \times 2 \times 2 \mathrm{X}-2 \times 2 \times 12-2 \times 12-12=0 \\
& \text { By rearranging we get }
\end{aligned}
$$

$$
\begin{equation*}
X=12(1 / 2+1 / 4+1 / 8)=84 / 8=10.5 \tag{1}
\end{equation*}
$$

In general, this may be written as:

$$
\begin{equation*}
\mathrm{X}=\mathrm{Y} 1 /(1+\mathrm{r})+\mathrm{Y} 2 /(1+\mathrm{r})^{2}+\mathrm{Y} 3 /(1+\mathrm{r})^{3} \tag{2}
\end{equation*}
$$

We can recognize that X is the present value but Fibonacci did not know that it was the present value and also did not understand its relevance. Why did Fibonacci not report the underlying equation? Charles Burnett (2005) believes that Fibonacci did not want to give credit to anyone so that he could claim as his own original contribution. He observes, "By the end of the twelfth century the 'algorismus' was widespread and it would certainly have been known to Fibonacci. In fact he refers specifically to the 'algorismus', in the preface to his Liber abbaci, as we have seen. But it is difficult to see why he should criticise it alongside the 'Gerbertian abacus', and state that the 'method of the Indians' is better, when the 'algorismus' is precisely 'the method of the Indians'. I suspect that his reason is that he thinks that, or wants to give the impression that, his own work is truly innovatory, for he scrupulously avoids mentioning the name of al-Khwarizmi. (In chapter 15 he uses only al-Khwarizmi's first name (ism), 'Maumeht'). Are we to suppose that Fibonacci was, in fact, ignorant of this Latin tradition of Indian arithmetic, and re-introduced the place-value numerals afresh from the Arabs?"
There could be another more compelling reason. Fibonacci dedicated the second edition of Liber Abaci to Michael Scott. Fibonacci in his Dedication and Prologue to his Liber Abaci wrote, "You, my Master Michael Scott [1], most great philosopher, wrote to my Lord [2] about the book on numbers which sometimes ago I composed and transcribed to you, whence complying with your criticisms, your more subtle examining circumspection, to the honor of you and many others I with advantage corrected this."
Michael Scott worked as an official astrologer to Emperor Frederick II and was
also in the good books of two popes, who recommended him for ` preferment. That is, he was closely connected to the Church. Fibonacci could not have afforded to displease Michael Scott or the two popes. As pointed out by Burnett, that Fibonacci 'scrupulously avoids mentioning the name of al-Khwarizmi'. But why? Raju (2013) offers a reason as to why Fibonacci did not want to acknowledge al-Khwarizimi’s work. Raju (p 129) points out, "The Crusades had aroused intense hatred of Muslims among Christians of Europe. So the priests who had so long been preaching Christian Supremacy felt a strong sense of shame in acknowledging the reality that Christians were learning from books written by Islamic scholars."
Problem \# 2: The Original Contract: Fibonacci (1202/2002, p 392):"A certain soldier because of his fief received from a certain king 300 bezants, and it is satisfied in IIII payments, and in each payment he takes 75 bezants; this is a payment for three months which by necessity is collected together."

Modified Contract: " $[\mathrm{H}] \mathrm{e}$ asks for a certain compensation in order to accommodate himself for interest because he accepts the 300 bezants instead of the 75 bezants of each payment, namely payment to payment, of capital and profit. Voluntarily acquiescing to this he invests the bezants at a profit of two bezants per hundred in each month. It is sought how many bezants he makes in his investment."
Fibonacci's Solution: " First indeed you strive to reduce this problem to the method of trips, and it is reduced thus; because in each month the profit from the 100 bezants is 2 bezants the profit for the one hundred is 6 bezants in the three months, namely at the time of each payment; therefore from each payment of 100 bezants is made 106, that is 53 is from 50 , and because there are IIII payments, IIII trips are similarly carried, and because the payment is 75 bezants, this is had for the expense of each trip."
Fibonacci thought that the modified contract was similar to the traveling trader's example. But they were different. The trader had some capital (10.5 Bezanets) when he started on the journey. On the other hand, the soldier had nothing. According to the original contract, the soldier received his first installment after 3 months and according to the modified contract, only interest payment at the end of second quarter. So Fibonacci should not have but still used the same technique to solve this problem, that is:

Balance after the first period= $\mathrm{X}(1+.06)-75$
Balance after the second period $=[\mathrm{X}(1+0.06)-75)](1+0.06)-75$
Balance after the third period $=[(\mathrm{X}(1+0.06)-75)(1+0.06)-75](1+.06)-75$
Balance after the fourth period ([(X $(1+0.06)-75)(1+0.06)-75](1+.06)-75))$
$(1+0.06)-75=0$, that is,

$$
\begin{aligned}
& X(1+0.06)^{4}-75(1+0.06)^{3}-75(1+0.06)^{2}-75(1+0.06)-75=0 \\
& 1.26247696 X=75(1.191016+1.1236+1.06+1)
\end{aligned}
$$

Solving for X gives
$X=7500(4.374616) / 1.26247696=259.882921$
Fibonacci, just like before, has the following equation in the background.

$$
\begin{align*}
& X=75\left(1 /(1+0.06)+1 /(1+0.06)^{2}+1 /(1+0.06)^{3}+1 /(1+0.06)^{4}\right) \\
& =259.882291 \tag{4}
\end{align*}
$$

Clearly, if discounting is not discernible in the first problem, it is neither in this problem.
Goetzmann claims, "The discounted annual value of the 300 bezants paid in the last period is 259 and change. As before, Fibonacci explains how to construct a multi-period discount factor from the product of the reciprocals of the periodic growth rate of an investment, using the model developed from mercantile trips in which a percentage profit is realized at each city. In this problem, he explicitly quantifies the difference in the value of two contracts due to the timing of the cash flows alone. As such, this particular example marks the discovery of one of the most important tools in the mathematics of Finance - an analysis explicitly ranking different cash flow streams based upon their present value."
Apparently, Goetzmann in his excitement jumped over some crucial details. All of his claims are false. First, his claim that the 'discounted value of 300 bezants is 259 and change' is not correct since the discounted value of 300 bezants is 237.628099 (Present value of 300 paid at the end $\left.=237.628099=300 /(1+0.06)^{4}\right)$ and not 259.882921).
Secondly, Goetzmann's claim that Fibonacci 'explicitly quantifies the difference in the value of two contracts due to the timing of the cash flows alone' as explained below, also is not correct.
Potential Value of the Original Contract: Apparently, the soldier wanted to invest his money as soon as he received it. In that case the potential value of his original contract, $\mathrm{X}_{0}$ would have been:

$$
\begin{aligned}
& X_{0}=75\left(1 /(1+0.06)+1 /(1+0.06)^{2}+1 /(1+0.06)^{3}+1 /(1+0.06)^{4}\right)+ \\
& 75(.06) /(1+0.06)^{2}+150(0.6) /(1+0.06)^{3}+225(0.6) /(1+0.06)^{4}
\end{aligned}
$$

$\mathrm{X}_{0}=259.882921$ (the discounted value of the capital) +22.254822 (discounted value of interest) $=282.137743$

Value of the Modified Contract: According to the modified contract, the soldier were to be paid interest on 75 bezants at a rate of 6 percent per quarter at the end of second quarter, that would be 4.5 bezants ( $75 \mathrm{x}(0.06)$ ), interest on 150 at the
end of third period, that would be 9 bezants (150x (0.06)) and interest of 13.5 ( $225 \mathrm{x}(0.06)$ ) bezants on 225 at the end of fourth period along with 300 bezants. We may write as:

Present value of interest payments=

$$
\begin{aligned}
& 75(.06) /(1+0.06)^{2}+150(0.6) /(1+0.06)^{3}+225(0.6) /(1+0.06)^{4} \\
& =4.5 /(1+0.06)^{2}+9 /(1+0.06)^{3}+13.5 /(1+0.06)^{4}
\end{aligned}
$$

$=22.254822$
Present value of 300 paid at the end $=237.628099=300 /(1+0.06)^{4}$
Present value of modified contract $=$ interest + capital $=22.254822+237.628099=$ 259.882921

Certainly, not only the respective cash flows of the original contract and that of the modified contract were different but also as shown above, the true value of the original contract was higher than that of the modified contract. However, Fibonacci did not quantify the difference between the original and the modified contracts and could not have done that.
Actually Fibonacci was not looking for a difference since he provided only one number ( 259.882921 ). He thought that the soldier was adequately compensated by allowing the interest payments and it did not matter whether the 300 bezanets were paid at the end or in four quarterly installments. He did not realize that the modified contract was less generous to the soldier than the original contract. It is clear that Fibonacci did not understand the concept of discounting or of present value.

## 4 Kautilya and Samuelson on Discounting for National Security Projects

Kautilya believed that a foreign ruler would ruin the economy. He (p 175) argued, "A foreign king, on the other hand, is one who has seized the kingdom from a legitimate king still alive; because it does not belong to him, he impoverishes it by extravagance, carries off its wealth or sells it. If the country becomes too difficult for him to handle, he abandons it and goes away (8.2)." He (132) added, "Harassment by the enemy's army [not only] affects the whole country but also ruins it by plunder, slaughter, burning and destruction (8.4)."
Kautilya also believed that a foreign ruler could cause instability. He (p 176) wrote, "A new king who has acquired the kingdom by his own might [usually] does as he pleases, as if it was his personal property. If he has been helped by others in the takeover, he has to tolerate them [even] if they oppress the country. [There is also the danger of instability] a usurper, with no firm roots among the people, is easily overthrown (8.2)."
Kautilya (p 179) suggested, "The king shall avoid [settling] any part of the country
which is liable to attack by enemies or jungle tribes and which is likely to be affected by disease and famine. He shall avoid excessive expenditure $(2,136)$."
Kautilya strongly recommended that before an enemy got a chance to ruin the economy and create instability, must be destroyed. He (p 541) wrote, "An enemy's destruction shall be brought about even at the cost of great losses in men, material and wealth (7.13)." If that was not possible, a king should try to acquire power to prevent an aggression. He developed a comprehensive approach to national security. His ideas can be represented by the following power equation (see Sihag (2014, Ch. 18) for details):

$$
\begin{align*}
& S=A(J, H)(K)^{\lambda}\left(E L_{m}\right)^{(1-\lambda)}  \tag{6}\\
& R=S_{1} / S_{2} \tag{7}
\end{align*}
$$

Where $S_{1}$ and $S_{2}=$ powers of king one and king two respectively, $A=$ efficiency parameter, $\mathrm{H}=$ experience and analytical skills of the advisers in utilizing the information made available through intelligence, $\mathrm{K}=$ horses, elephants, chariots and armaments, $\mathrm{E}=$ enthusiasm and training, $\mathrm{L}_{\mathrm{m}}=$ military strength, $\mathrm{J}=$ level of public support for a just and kind-hearted king and $\mathrm{R}=$ first king's relative power. Thus, according to Kautilya, (i) probability of an aggression was negatively related to the relative power and there was always a potential threat to a nation so long as its relative power was less than one. (ii) A foreign ruler was interested in promoting only his own well-being and could be a source of political instability and (iii) instability would lead to a higher risk premium. ${ }^{6}$
Kautilya's advice to the king was to take precautionary measures to reduce the probability of any man-made or natural disaster and if despite preventive measures, it still happened, remedial measures should be in place to reduce the potential loss. He ( p 116 ) wrote, "In the interests of the prosperity of the country, a king should be diligent in foreseeing the possibility of calamities, try to avert them before they arise, overcome those which happen, remove all obstructions to economic activity

[^4]and prevent loss of revenue to the state (8.4)." It may be noted that, according to Kautilya, $p$ the probability of an attack is a policy variable.
Incidentally, Ehrlich and Becker (1972) label 'try to avert them before they arise' as self-protection and 'overcome those which happen' as self-insurance. Equation (8) in Barro and Urs'ua (2012) may be used to express Kautilya's ideas:
\[

$$
\begin{equation*}
r e-r f=\gamma \sigma 2+p \cdot E\{b \cdot[(1-b)-\gamma-1]\} \tag{8}
\end{equation*}
$$

\]

Where $r e=$ is the expected rate of return on equity, and $r f=$ is the risk-free rate. Kautilya argued that the risk premium ( $r e-r f$ ) could be lowered by reducing the probability, p of a disaster, such as a war. ${ }^{7}$
Similarly, according to Samuelson, national security projects, such as nuclear treaties are very different from other projects since they lower the probability of a war and consequently lower the risk premium for all ventures/stocks. Additionally, Samuelson suggests that fiscal policies could be used to reduce the probability of a depression and also the loss of income. He (1964, p 95) wrote, "A private investor faces depression risk in every venture. It would be a blessing if the government incurred dollar losses at such times. Only those huge decisions that bring on serious results to all citizens-atomic bombs treaties, and not regional dams-need strong discounting for risk dispersion." Jensen and Bailey (1972) did not understand the importance of national security. They wrote, "Thus, Samuelson notwithstanding, it is precisely "regional dams" and not "atomic bomb treaties" that are more likely to need "strong discounting for risk" when undertaken in the public sector."

## 5 Conclusion

Kautilya was the first economist, who introduced the concept of discounting. He needed this concept along with other concepts to give shape to his vision of building a prosperous, secular, safe and secure nation. He introduced the concept of risk premium, particularly associated with instability/insecurity caused by a foreign aggressor or a local intruder. Most importantly, he justified giving preference to projects that enhanced national security. On the other hand as shown above, Fibonacci did not understand the concept of discounting. He was simply trying to hide his sources by using sentences instead of an equation.
Grampp (2000) is concerned about the possibility of excessive imagination on the part of some researchers in interpreting earlier writers. Accordingly, he lays some

[^5]ground rules to avoid such tendencies. ${ }^{8}$ Any claim, that Fibonacci even remotely could have conceived the concept of discounting, would fall under Grampp's (e) and (f) categories. Also there is no evidence to support the claim that Fibonacci created a financial revolution. ${ }^{9}$ Europe was in a deep slumber at least for another three hundred years and Fibonacci's Liber Abaci was unsuccessful in waking it up.

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${ }^{8}$ Grampp (2000) proposes: "A way to get things straight about Smith or anyone else. It is to begin by distinguishing between (a) what the author actually said, (b) what is implied by what he said, (c) what can reasonably be inferred from it, (d) what we may conjecture he meant, (e) what he conceivably could have meant, and (f) what it would be convenient to believe what he meant. The next step is to stay as close as possible to points (a) and (b), to know that about point (c), the operative word is "reasonably," and to move as far as point (d) only when all else fails, or never at all. Distinctions (e) and (f) are left to those who, to paraphrase George Stigler, make the study of economic ideas a work of the imagination."
${ }^{9}$ Peter Bernstein (1996, p 18) "Up to the time of the Renaissance, people perceived the future as little more than a matter of luck or the result of random variations, and most of their decisions were driven by instinct. When the conditions of life are so closely linked to nature, not much is left to human control. As long as the demands of survival limit people to the basic functions of bearing children, growing crops, hunting, fishing, and providing shelter, they are simply unable to conceive of circumstances in which they might be able to influence the outcomes of their decisions. A penny saved is not a penny earned unless the future is something more than a black hole." Similarly, O'Connor and Robertson (2001) remark, The first surviving example of the Indian numerals in a document in Europe was, however, long before the time of al-Banna. The numerals appear in the Codex Vigilanus copied by a monk in Spain in 976. However the main part of Europe was not ready at this time to accept new ideas of any kind. Acceptance was slow, even as late as the fifteenth century when European mathematics began its rapid development which continues today.

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[^1]:    ${ }^{2}$ The following table lists some, other than discounting, concepts innovated and used (see Sihag (2014) for applications) by Kautilya. It also provides the time-periods of their re-emergence.

[^2]:    ${ }^{3}$ Mattessich (2000, p 203-204) remarks, "Kautilya's Arthashastra is not merely significant for only for business accounting but also government accounting, with some stretch of imagination it may be regarded as a forerunner of national income accounting since the ultimate purpose of Kautilya's work was to strengthen the economy of the entire nation. Its significance lies in the attempt to offer accounting concepts of fairly general validity...;This treatise may even be called in evidence for the close relationship between micro- and macro-accounting concepts."
    ${ }^{4}$ Sihag (2014, Chap 5) on Kautilya's ideas on evaluation of a policy/project.
    "Kautilya (p 544) stated, "Events, both human and providential, govern the world (and its affairs). Acts of God are those which are unforeseeable and whose origin is unknown. If the cause is knowable and, hence, foreseeable, its origin is human. If an act of God results in (helping) the achievement of one's objective, it is good fortune; otherwise, it is misfortune. (Likewise,) any human action which increases one's wealth is a good policy; otherwise, it is a bad policy." Analysis of Variance: The above statement amounts to the specification of a regression model and separating the total variation (in the dependent variable, wealth) into explained and unexplained (random) components. This may formally be specified as follows:
    $\mathrm{W}=\mathrm{X} \beta+\varepsilon$ (3), Where, $\mathrm{W}=$ wealth, the dependent variable, $\mathrm{X}=$ exogenous policy variables, which may be used to acquire wealth, $\varepsilon=$ random error (the acts of God). Incidentally, the most important assumption of regression analysis that the error terms are independent of the right hand side variables, X , that is, $\operatorname{Cov}(\mathrm{X}, \varepsilon)=0$, is clearly satisfied in the above statement. Additionally, the Covariance $\left(\varepsilon_{\mathrm{t}}, \varepsilon_{\mathrm{t}-1}\right)=0$ since acts of God are likely to be independent of each other across different years. It is not claimed that he knew the implications of these assumptions."

[^3]:    ${ }^{5}$ Fibonacci's Other Trial and Error Method to find the Solution: He (P 460) explains the other method as: "You indeed put it that he has 12 denari of which he made double in the first trip, and he thus had 24 denari from which he spent 12 denari and thus remained for him another 12 denari, of which he made double in the remaining two trips and he spent in each 12 denari: there remained for him at the end 12 denari. Therefore, in the position I erred in value by plus 12; therefore you put it that he had 11 denari from which, as he made double in the three trips and spent in each 12 , there remained for him at the end 4 denari, namely 8 fewer than in the first position. And therefore this position is too large. When you say: for the 1 which I decreased in the capital I approximated more closely by 8 how many shall I decrease again so that the approximation is decreased 4 further? You therefore multiply the 4 by the 1 , and you divide by the 8 , the quotient will be $1 / 2$ of one denaro; this subtracted from the 11 denari leave (1/2)10 denari for the capital. Or from the multiplication of the first error by the second position, namely 132 , you subtract the 48 that results from multiplying the second error by the first position leaving 84 which divided by the difference in errors leaves (1/2)10."

[^4]:    ${ }^{6}$ Kautilya linked risk premium to instability. He (p 426) suggested, "The lawful rates of interest (on money lent) for different purposes shall be:
    Normal transactions $15 \%$ per annum,
    Commercial transactions: Normal $60 \%$
    Risky travel:
    Through forests $120 \%$
    By sea $240 \%$
    No one shall charge or cause to be charged a rate higher than the above, except in regions where the King is unable to guarantee security; in such a case, the judges shall take into account the customary practices among debtors and creditors (3.11)." His suggestion that interest rate could be higher 'where the King is unable to guarantee security' is quite significant, implying the importance of law and order in reducing risk and promoting economic activity (see Sihag (2013) for more details).

[^5]:    ${ }^{7}$ Barro (2006) defines probability of a disaster as: "Actual and potential economic disasters could reflect economic events (the Great Depression, financial crises), wartime destruction (world wars, nuclear conflicts), natural disasters (tsunamis, hurricanes, earthquakes, asteroid collisions), and epidemics of disease (Black Death, avian flu). I use the twentieth-century history of economic disasters to assess disaster probabilities, sizes of contractions, and default probabilities."

