

The Chaotic Agricultural Production Growth Model: EU

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Abstract

Economic welfare depends on the amount and stability of agricultural production. The stability of agricultural production assumes a major strategic and geopolitical character. The basic aim of this analysis is to set up a relatively simple chaotic agricultural production growth model that is capable of generating stable equilibria, cycles, or chaos. It is important to analyse the stability of the agricultural production growth in the EU in the period 1996-2012. This paper finds out the existence of the convergent fluctuations of agricultural production in the EU in the period 1996-2012.

JEL classification numbers: O13, H62, C2

Keywords: Agriculture, Budget deficit, Single equation model

1 Introduction

The basic aims of this analysis is: firstly, to set up a relatively simple chaotic agricultural production growth model that is capable of generating stable equilibria, cycles, or chaos; and secondly, to analyse the stability of the agricultural production growth in the EU in the period 1996-2012 (see Fig 1.).

Productive agriculture brings significant benefits for food security, resource efficiency and economic stability. Global economic crisis developed with remarkable speed starting in the late summer of 2008. A typical financial crisis is described by declines in asset prices and the failures of financial system. Government budget deficit raises real interest rates. When the government spends more money than it spends, then the government runs a budget deficit and public saving becomes a negative number (see Fig 2.).

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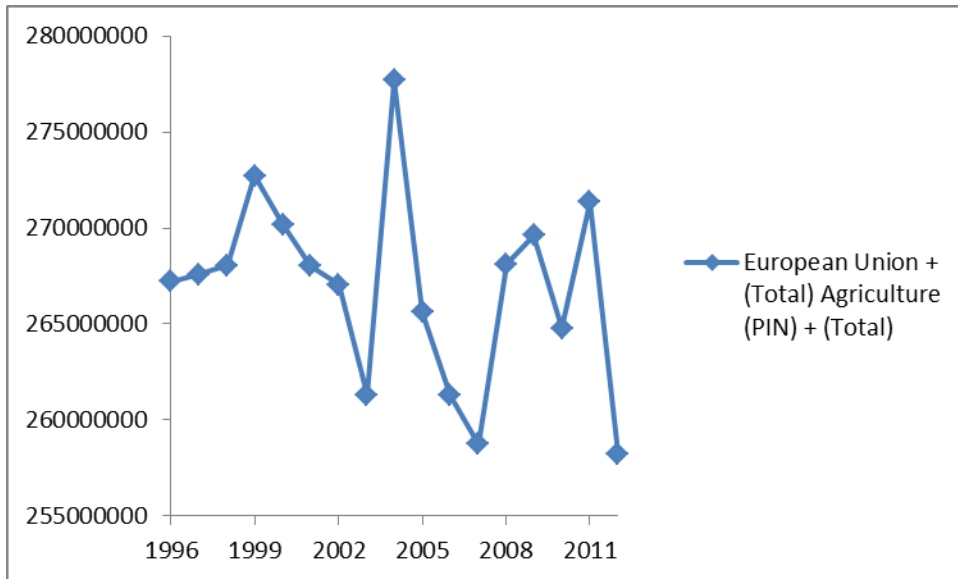


Figure 1: Gross Agricultural Production Value (constant 2004-2006 1000 I\$): EU, 1996-2012 Source: www.fao.org

Economic systems are inherently acyclical. In this context, it is important to analyse the stability of the agricultural production growth by nonlinear, chaotic method. The application of this method proves detailed results.

In 2012, agriculture in the EU-27 generated around EUR 159.4 billion of value added, some 1.4% of the added value for the whole economy: the contribution of agriculture fell from 1.8% a decade earlier (2002), to a low of 1.2% in 2009, before increasing each year through to 2012. The economic importance of agriculture, in value added terms, was generally much greater in the east and south of Europe than in the west and north. Agriculture is a highly labour-intensive activity. Agricultural labour productivity can be influenced by factors such as average farm sizes, the level of mechanisation and the share of production for on-farm consumption.

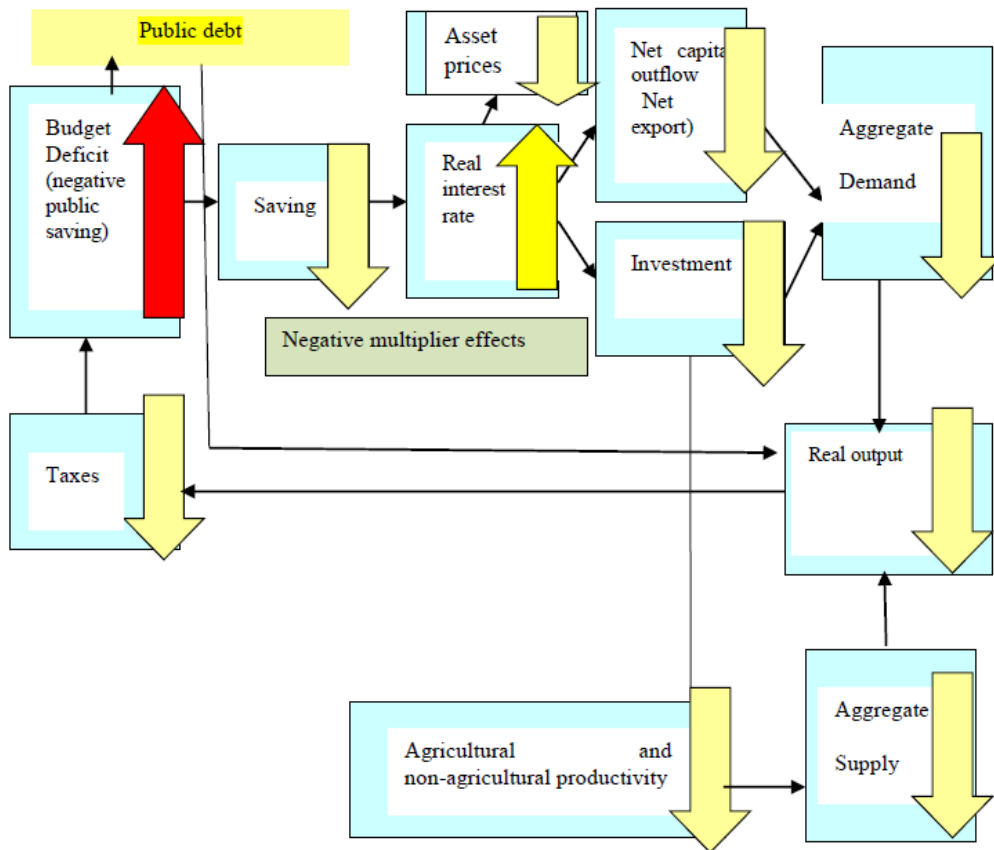


Figure 2: Causes of the financial crisis or the effects of a government budget deficit and public debt: When the government runs a budget deficit, it reduces the national saving. The interest rate rises. The asset prices decrease. Further, the higher interest rate reduces net foreign investment (net capital outflow). Reduced net foreign investment (net capital outflow), in turn, reduces the supply of domestic currency in the market for foreign-currency exchange, which causes the real exchange rate of domestic currency to appreciate. Agricultural and non-agricultural productivity decrease and aggregate supply decreases.

Yang and Zhu (2013) develop a two-sector model that illustrates the role played by agricultural modernization in the transition from stagnation to growth. In the case of traditional technology, industrial development reduces the relative price of industrial products, but has a limited effect on per capita income. Growth is not sustainable until this relative price drops below a certain limit. Then, farmers adopt modern technology that employs industry-supplied inputs. Further, per capita income accelerates toward modern growth. Fan et al. (2013) provide their own perspectives on the China’s economic development over the past 35 years. Agriculture has important role. Lagakos and Waugh (2013) explain that cross-country labor productivity differences are larger in agriculture than in non-agriculture and suggest a new explanation for these patterns in which the self-selection of heterogeneous workers determines sector productivity. They set up a general-equilibrium Roy model which predict that productivity differences are roughly twice as

large in agriculture as non-agriculture. Bates and Block (2013) use a recent analysis of total factor productivity growth in African agriculture. They find that the introduction of competitive presidential elections in the last decades of the twentieth century appears to have altered political incentives on agricultural growth.

Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change.

Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981, 1982), Day (1982, 1983), Lorenz (1993), Jablanovic (2012a, 2012b, 2013), among many others.

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst, a Belgian mathematician interested in the modeling of human populations. He applied his logistic equation for population growth to demographic studies. The model was created to produce solutions for a single population that grows from a small initial number of individuals to a limited population with an upper bound implicitly set by factors such as food supply, living space or pollution (the effect of a "carrying capacity" that would limit growth). His formula is called the "logistic model" or the Verhulst model.

Linear analysis used in the theory of agricultural growth presumes an orderly periodicity that rarely occurs in economy. In this sense, it is important to construct deterministic, nonlinear, chaotic model that elucidate irregular, unpredictable economic behaviour.

The suggested application of chaotic method consists of six phases: (i) observe of a economic phenomenon (agricultural production); (ii) set up the hypothesis to explain the movement of the agricultural production; (iii) create the chaotic model; (iv) analyse the local stability of the agricultural production in the EU in the observed period; (v) obtain detailed results (*the convergent* fluctuations of the agricultural production), and (vi) set out the policy recommendations arising from the research.

2 The Model

National saving is the source of the supply of loanable funds. Domestic investment and net capital outflow are the sources of the demand for loanable funds. At the equilibrium interest rate, the amount that people want to save exactly balances the amount that people want to borrow for the purpose of buying domestic capital and foreign capital. At the equilibrium

interest rate, the amount that people want to save exactly balances the desired quantities of domestic investment and net capital outflow. Further, national saving can be separated into two pieces: private saving and public saving. Budget deficit represents negative public saving. Increasing domestic investment assumes increasing agricultural and non-agricultural productivity. On the other hand, when a government budget deficit crowds out investment, it reduces the growth of the agricultural and non-agricultural productivity and the total and agricultural output.

The chaotic agricultural production growth model can be described by the following equations :

$$S_t = I_t + NCO_t \quad (1)$$

$$S_t = S_{p,t} - Bd_t \quad (2)$$

$$NCO_t = n Y_t \quad (3)$$

$$S_{p,t} = s S_t \quad (4)$$

$$I_t = i Y_t \quad (5)$$

$$Bd_t = b Y_t \quad (6)$$

$$\frac{I_{t+1} - I_t}{I_t} = \alpha - \beta Bd_t \quad (7)$$

$$Y_{a,t} = a Y_t \quad (8)$$

Where : S – national saving; Y- total output ; Y_a- agricultural output ; NCO – net capital outflow (net foreign investment); I- investment; S_p – private saving; Bd – budget deficit; n - net capital outflow as a percent of the total output ; b – budget deficit rate; s – private saving as a percent of the national saving; α – an autonomous investment growth rate ; β – the coefficient which explains relation between an investment growth rate and budget deficit ; a- the share of agricultural production in total output; i- an investment rate.

(1) shows that national saving equals private saving plus net capital outflow; (2) shows that national saving equals private saving minus budget deficit as a negative public saving; (3) describes the relation between net capital outflow and the gross domestic product (total real output); (4) shows that private saving depends on national saving; (5) states that investment depends on the gross domestic product (total real output); (6) shows that budget deficit depends on the gross domestic product; (7) shows that a government budget deficit crowds out investment (»crowding out effect«); (8) shows that agricultural output depends on the gross domestic product (total real output).

By substitution one derives:

$$Y_{a,t+1} = \left[\frac{(1+\alpha)(s-b)}{\beta} \right] Y_{a,t} - \left(\frac{bi}{a} \right) Y_{a,t}^2 \quad (9)$$

Further, it is assumed that the current value of the agricultural production is restricted by its maximal value in its time series. This premise requires a modification of the growth law.

Now, the agricultural production growth rate depends on the current size of the agricultural production, Y_a , relative to its maximal size in its time series Y_a^m . We introduce y_a as $y_a = Y_a / Y_a^m$. Thus y range between 0 and 1. Again we index y by t , i.e., write $y_{a,t}$ to refer to the size at time steps $t = 0, 1, 2, 3, \dots$. Now growth rate of the agricultural production is measured as:

$$y_{a,t+1} = \left[\frac{(1+\alpha)(s-b)}{\beta} \right] y_{a,t} - \left(\frac{bi}{a} \right) y_{a,t}^2 \quad (10)$$

This model given by equation (10) is called the logistic model. For most choices of α , β , a , s , i , and b there is no explicit solution for (10). This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (10) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behaviour of y_t . This difference equation (10) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point $y_{a,0}$ the solution is highly sensitive to variations of the parameters α , β , a , s , i , and b ; secondly, given the parameters α , β , a , s , i , and b , the solution is highly sensitive to variations of the initial point $y_{a,0}$. In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t) \quad , \quad \pi \in [0, 4] \quad , \quad z_t \in [0, 1] \quad (11)$$

is equivalent to the iteration of growth model (12) when we use the identification

$$z_t = \frac{\beta b i}{a (1 + \alpha) (s - b)} y_{a,t} \quad (12)$$

and

$$\pi = \left[\frac{(1 + \alpha) (s - b)}{\beta} \right] \quad (13)$$

Using (12) and (10) we obtain

$$\begin{aligned} z_{t+1} &= \frac{\beta b i}{a (1 + \alpha) (s - b)} y_{a,t+1} = \\ &= \left[\frac{\beta b i}{a (1 + \alpha) (s - b)} \right] \left\{ \left[\frac{(1 + \alpha) (s - b)}{\beta} \right] y_{a,t} - \left(\frac{b i}{a} \right) y_{a,t}^2 \right\} \end{aligned}$$

$$= \left\{ \left(\frac{b i}{\alpha} \right) y_{a,t} - \left[\frac{\beta b^2 i^2}{\alpha a (1+\alpha) (s-b)} \right] y_{a,t}^2 \right\}$$

On the other hand, using (11) , (12) and (13) we obtain

$$\begin{aligned} z_{t+1} &= \pi z_t (1 - z_t) = \\ &= \left[\frac{(1+\alpha)(s-b)}{\beta} \right] \left[\frac{\beta b i}{a (1+\alpha) (s-b)} \right] y_{a,t} \left\{ 1 - \left[\frac{\beta b i}{a (1+\alpha) (s-b)} \right] y_{a,t} \right\} \\ &= \left\{ \left(\frac{b i}{\alpha} \right) y_{a,t} - \left[\frac{\beta b^2 i^2}{\alpha a (1+\alpha) (s-b)} \right] y_{a,t}^2 \right\} \end{aligned}$$

Thus we have that iterating (10) is really the same as iterating $z_{t+1} = \pi z_t (1 - z_t)$ using (12) and (13).

It is important because the dynamic properties of the logistic equation (12) have been widely analysed (Li and Yorke (1975), May (1976)).

It is obtained that :

- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (ii) For $1 < \pi < 3,57$ there exist fixed points the number of which depends on π ;
- (iii) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;
- (iv) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- (v) For $3 < \pi < 4$ all solutions will continuously fluctuate;
- (vi) For $3,57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

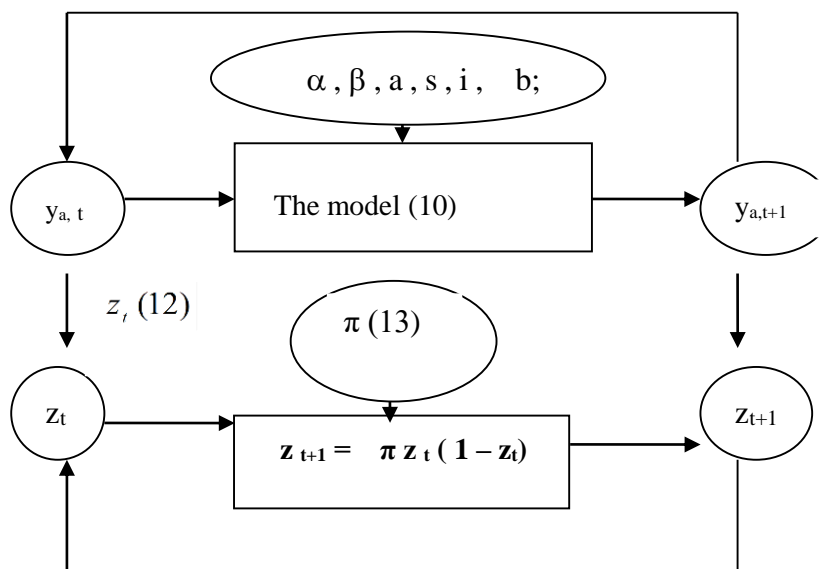


Figure 3: Two quadratic iterators running in phase are tightly coupled by the transformations indicated

3 Main Results

The main aim of this paper is to analyse the agricultural production growth stability in the period 1996-2012, in the EU by using the presented non-linear, logistic agricultural production growth model (10). However, we rewrite the difference equation (10) on the following way, i.e.,

$$y_{a,t+1} = \pi y_{a,t} - \theta y_{a,t}^2 \quad (14)$$

where: y_a – agricultural production; $\pi = (1+\alpha)(s-b)/\beta$; $\theta = \beta i / \alpha$

Firstly, data on the agricultural production are transformed (Source: www.fao.org) from 0 to 1, according to our supposition that actual value of the agricultural production, Y_a , is restricted by its highest value in the time-series, Y_a^m . Further, we obtain time-series of $y_a = Y_a / Y_a^m$

Table 1: The estimated model (14): EU, 1996-2012 (Source: www.fao.org)

	π	θ
Estimate	2.15376	1.199820
Std.Err.	.295545	.306651
t(14)	7.287457	3.912660
p-level	0.00000	0.001562

4 Conclusion

This paper creates the simple chaotic agricultural production growth model. The model (10) has to rely on specified parameters α , β , a , s , i , and b , and initial value of the agricultural production, y_{a0} . This difference equation (10) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point y_{a0} the solution is highly sensitive to variations of the parameters α , β , a , s , i , and b ; secondly, given the parameters α , β , a , s , i , and b , the solution is highly sensitive to variations of the initial point y_{a0} . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

A key hypothesis of this work is based on the idea that the coefficient $\pi = (1+\alpha)(s-b)/\beta$ plays a crucial role in explaining local agricultural production growth stability where: b – budget deficit rate; s – private saving as a percent of the national saving; α – an autonomous investment growth rate; β – the coefficient which explains relation between an investment growth rate and budget deficit.

The estimated value of the coefficient π is 2.15376 (see Table 1). This result confirms stable *convergent fluctuations* of the agricultural production in the EU in the period 1996-2012. Stable agricultural production supposes important benefits to the EU, from increased food security to economic stability. Several strategies for stabilizing the rate of agricultural growth in EU have been suggested. The chaotic model (10) suggests the strategies which include policies aimed toward increasing the saving rate, stimulating investment in physical capital, stimulating investment in human capital, stimulating investment in the creation and dissemination of technological knowledge, decreasing budget deficit, and pursuing a restrictive fiscal policy. The amount of capital accumulation in an economy depends on its rate of national saving. On the other hand, budget deficit as a negative public saving, reduces agricultural and non-agricultural investment.

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