

The Nonlinear Real Interest Rate Growth Model : USA

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Abstract

The article focuses on the chaotic real interest rate growth model. According to the classical theory, the interest rate is determined by the intersection of the investment demand-schedule and the saving-schedule. However, this model supposed that the real interest rate is determined by the intersection of the investment demand-schedule and net capital outflow-schedule and the saving-schedule. The basic aim of this analysis is to set up a relatively simple chaotic real interest rate growth model that is capable of generating stable equilibria, cycles, or chaos. It is important to analyze the stability of the real interest growth in USA in the following periods: 1989-1993, 1998-2004, and 2007-2011. A key hypothesis of this work is based on the idea that the coefficient $\pi = b / g+e$ plays a crucial role in explaining local growth stability of the real interest rate, where, b - the coefficient of the investment function, g - the coefficient of the net capital outflow function, e - the coefficient of the saving function. The estimated values of the coefficient π are smaller than 1 in the observed periods. This result confirms decreasing real interest movement in USA in the observed periods.

JEL classification numbers: E43, E21, E22, F21.

Keywords: Interest Rate, Saving, Investment, Net capital outflow.

1 Introduction

Loanable funds is the sum total of all the money people in an economy has decided to save and lend out to borrowers as domestic investment and net capital outflow .The supply of loanable funds comes from national saving, including both private saving and public saving. The supply of loanable funds comes from those people who have decided not to spend some of their money but instead save it for investment purposes. One way to make an investment is to lend money to borrowers at a rate of interest. This national saving, including both private saving and public saving, makes up the supply of loanable funds. The demand for loanable funds comes from firms and households that want to borrow for

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purposes of (domestic or foreign) investment. To keep things simple, we use a version classical market analysis to describe the supply, demand and interest rates for loans in the market for loanable funds. Namely, the law of supply and demand is applicable in the market for loanable funds. The real interest rate in the economy adjusts to balance the supply and demand for loanable funds. According to the classical theory, the interest rate is determined by the intersection of the investment demand-schedule and the saving-schedule. However, this model supposed that the real interest rate is determined by the intersection of the investment demand-schedule and net capital outflow-schedule and the saving-schedule.

As the interest rate on loanable funds increases, the quantity of funds demanded will decrease. On the other hand, as the interest rate for loanable funds increase, the supply of loanable funds also increases. In this sense, the adjustment of the interest rate coordinates the the behavior of the suppliers of loanable funds and the behavior of the demanders of loanable funds. The real interest rate for loanable funds will reach equilibrium where demand for loanable funds equal the supply of loanable funds offered for investment.

The following figure depicts the loanable fund market at equilibrium where S , national saving, equals supply, I equals investment, NCO equals net capital outflow, and r is the rate of real interest. Put the rate of real interest on the vertical axis. Put the flows of saving, investment and net capital outflow on the horizontal axis. Draw a downward-sloping desired investment curve I . Draw a downward-sloping desired net capital outflow curve, NCO . Draw an upward-sloping desired saving curve S . Mark the point where the curves cross. Draw a horizontal line to the vertical axis, to find the equilibrium rate of real interest. The rate of interest is determined by the intersection of the desired saving and desired investment plus net capital outflow curves.

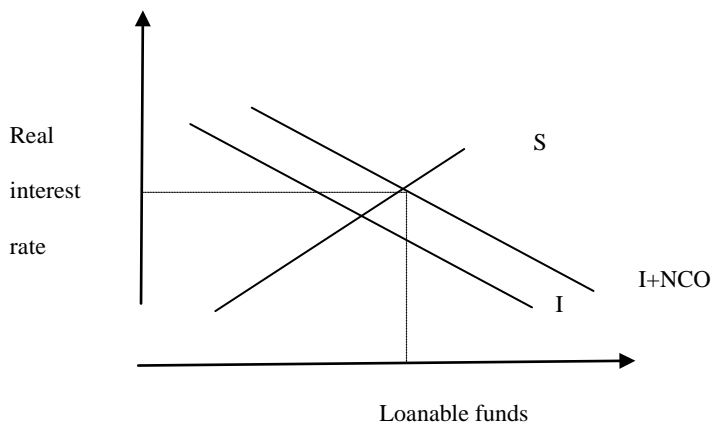


Figure 1: The Market for Loanable Funds

In an open economy, government budget deficit, as a negative public saving, raises real interest rates, crowds out domestic investment, decreases net capital outflow, decreases the level of asset prices, causes the domestic currency to appreciate. A decrease in aggregate demand causes output and prices to fall. The recession may further increase budget deficit and public debt.

National saving is the source of the supply of loanable funds. Domestic investment and net capital outflow are the sources of the demand for loanable funds. At the equilibrium real interest rate, the amount that people want to save exactly balances the amount that people

want to borrow for the purpose of buying domestic capital and foreign capital. At the equilibrium interest rate, the amount that people want to save exactly balances the desired quantities of domestic investment and net capital outflow.

The supply and demand for loanable funds determine the real interest rate. The interest rate determine net capital outflow, which provides the supply of dollars in the market for foreign –currency exchange. The supply and demand for dollars in the market for foreign-currency exchange determine the real exchange rate.

In an open economy, government budget deficit raises real interest rates, crowds out domestic investment, decreases net capital outflow, decreases the productivity growth, decreases the level of asset prices, causes the domestic currency to appreciate. However this appreciation makes domestic goods and services more expensive compared to foreign goods and services. In this case, exports fall, and imports rise. Namely, net exports fall. Further, the real gross domestic product falls as a consequence of the negative open economy multiplier effects

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In this sense it is important to estimate the saving function and the investment function. The estimated saving function (USA, 1989-2013) is presented in Table 1. The estimated investment function (USA, 1989-2013) is presented in Table 2.

Table 1: The estimated saving function ($S_t = d + e r_t$) : USA, 1989-2013

R=.82128 Variance explained: 67.451%		
	d	e
Estimate	0.14876	0.750899
Std.Err.	0.00509	0.108766
t(23)	29.21829	6.903777
p-level	0.00000	0.00000

Source: (www.imf.org) and (<http://data.worldbank.org/indicator/FR.INR.RINR>)

Table 2: The estimated investment function ($I_t = a + b r_{t-1} - c r_{t-1}^2$) : USA, 1989-2013.

R=.65933 Variance explained: 43.471%			
	a	b	c
Estimate	0.16812	1.946378	17.13817
Std.Err.	0.01480	0.815654	9.56899
t(21)	11.35633	2.386278	1.7910
p-level	0.00000	0.026511	0.08772

Source: (www.imf.org) and (<http://data.worldbank.org/indicator/FR.INR.RINR>)

Chaos theory is used to prove that erratic and chaotic fluctuations can indeed arise in completely deterministic models. Chaos theory reveals structure in aperiodic, dynamic systems. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change.

Chaos theory started with Lorenz's [1963] discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke [1975] discovered that the simple logistic curve can exhibit very complex behaviour. Further, May [1976] described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day [1981, 1982], Day [1982, 1983], Grandmont [1985], Goodwin [1990], Medio [1993], Lorenz [1993], Jablanovic [2012, 2014a, 2014b], Puu, T. [2003], Zhang W.B. [2006], among many others.

2 The Model

Financial markets are like other markets. The price of loanable funds, the real interest rate, is governed by the forces of supply (S_t) and demand ($I_t + NCO_t$). In this sense, the chaotic real interest rate growth model can be described by the following equations :

$$S_t = I_t + NCO_t \quad (1)$$

$$I_t = a + b r_{t-1} - c r_{t-1}^2 \quad (2)$$

$$S_t = d + e r_t \quad (3)$$

$$NCO_t = f - g r_t \quad (4)$$

Where : S – national saving; Y - the real gross domestic product; NCO – net capital outflow (net foreign investment); I - investment; r – the real interest rate; a, b, c – the coefficients of the investment function; d, e – the coefficients of the saving function, f, g – the coefficients of the net capital outflow function.

It is supposed that a, d , and f are zero. By substitution one derives:

$$r_t = \left(\frac{b}{g+e} \right) r_{t-1} - \left(\frac{c}{g+e} \right) r_{t-1}^2 \quad (5)$$

Further, it is assumed that the current value of the real interest rate is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the real interest rate growth rate depends on the current size of the real interest rate, r , relative to its maximal size in its time series r^m . We introduce y as $i = r / r^m$. Thus i range between 0 and 1. Again we index i by t , i.e., write i_t to refer to the size at time steps $t = 0, 1, 2, 3, \dots$ Now growth rate of the real interest rate is measured as:

$$i_t = \left(\frac{b}{g + e} \right) i_{t-1} - \left(\frac{c}{g + e} \right) i_{t-1}^2 \tag{6}$$

This model given by equation (6) is called the logistic model. For most choices of b , c , g , and e there is no explicit solution for (6). This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (6) can lead to very interesting dynamic behavior, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of i_t . This difference equation (6) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point i_0 the solution is highly sensitive to variations of the parameters b , c , g , and e ; secondly, given the parameters b , c , g , and e , the solution is highly sensitive to variations of the initial point i_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

3 The Logistic Model

The logistic map is often cited as an example of how complex, chaotic behavior can arise from very simple non-linear dynamical equations. The logistic model was originally introduced as a demographic model by Pierre François Verhulst, who applied his logistic equation for population growth to demographic studies. This model is called the "logistic model" or the Verhulst model.

It is possible to show that iteration process for the logistic equation (see Fig.2.)

$$z_t = \pi z_{t-1} (1 - z_{t-1}) \quad , \quad \pi \in [0, 4] \quad , \quad z_t \in [0, 1] \tag{7}$$

is equivalent to the iteration of growth model (6) when we use the identification

$$z_t = \frac{c}{b} i_t \tag{8}$$

and

$$\pi = \frac{b}{g + e} \tag{9}$$

Using (6) and (8) we obtain

$$z_t = \frac{c}{b} i_t = \left(\frac{c}{b} \right) \left[\left(\frac{b}{g + e} \right) i_{t-1} - \left(\frac{c}{g + e} \right) i_{t-1}^2 \right] =$$

$$= \left(\frac{c}{g+e} \right) i_{t-1} - \left[\frac{c^2}{b(g+e)} i_{t-1}^2 \right]$$

On the other hand, using (7), (8) and (9) we obtain

$$\begin{aligned} z_t &= \pi z_{t-1} (1 - z_{t-1}) = \\ &= \left(\frac{b}{g+e} \right) \left(\frac{c}{b} \right) i_{t-1} \left[1 - \left(\frac{c}{b} \right) i_{t-1} \right] \\ &= \left(\frac{c}{g+e} \right) i_{t-1} - \left[\frac{c^2}{b(g+e)} i_{t-1}^2 \right] \end{aligned}$$

Thus we have that iterating (6) is really the same as iterating (7) using (8) and (9).

It is important because the dynamic properties of the logistic equation (7) have been widely analyzed (Li and Yorke [1975], May [1976]).

It is obtained that :

- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- For $1 < \pi < 3,57$ there exist fixed points the number of which depends on π ;
- For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;
- For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- For $3 < \pi < 4$ all solutions will continuously fluctuate;
- For $3,57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

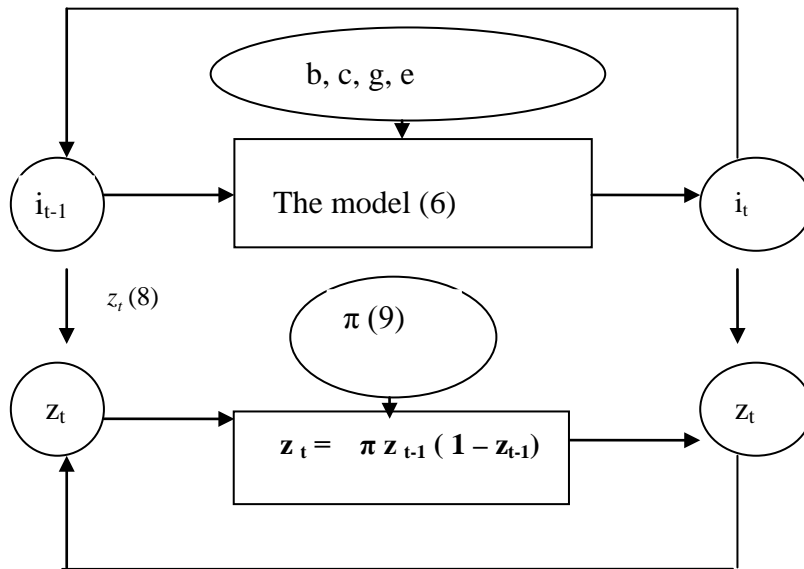


Figure 2: Two quadratic iterations running in phase are tightly coupled by the transformations indicated

4 Empirical Evidence

The main aim of this paper is to analyze the real interest growth rate stability (see Fig. 3) in the following periods: 1989-1993, 1998-2004, and 2007-2011., in USA by using the presented non-linear real interest rate growth model (10) :

$$i_t = \pi i_{t-1} - \beta i_{t-1}^2 \tag{10}$$

where: i – the real interest rate, $\pi = b/g+e$, $\beta = c/g+e$.

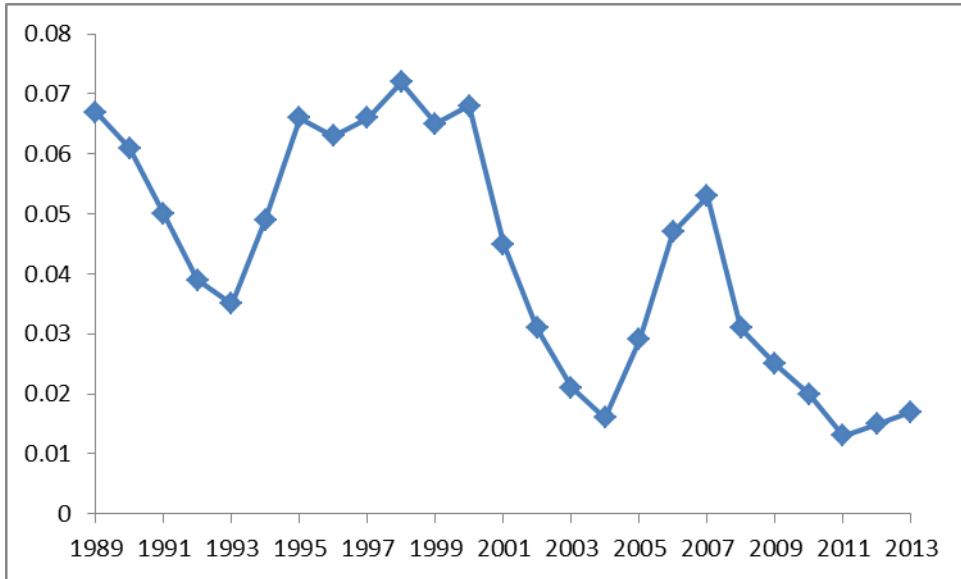


Figure 3: The real interest rate in USA,1989-2013. Source:
(<http://data.worldbank.org/indicator/FR.INR.RINR>)

Firstly, data on the real interest rate are transformed from 0 to 1, according to our supposition that actual value of the real interest rate, r , is restricted by its highest value in the time-series, r^m . Further, we obtain time-series of $i = r / r^m$.

(<http://data.worldbank.org/indicator/FR.INR.RINR>)

Table 3: The estimated model (10)

1989-1993	R=.96130 Variance explained: 92.410%	
	π	β
Estimate	0.742883	-1.91808
Std.Err.	0.2215	3.75494
t(3)	3.35387	-0.51082
p-level	0.078567	0.66028
1998-2004	R=.92169 Variance explained: 84.952 %	
	π	β
Estimate	0.570324	-4.19608
Std.Err.	0.355928	5.56885
t(5)	1.602358	-0.75349
p-level	0.184336	0.49308
2007-2011	R= .92093 Variance explained: 84.812%	
	π	β
Estimate	0.997568	0.674571
Std.Err.	0.149408	0.298465
t(3)	0.676825	2.260134
p-level	0.006851	0.108912

Source: (<http://data.worldbank.org/indicator/FR.INR.RINR>)

5 Conclusion

This paper creates the simple chaotic real interest rate growth model . The model (6) has to rely on specified parameters b , c , g , and e , and initial value of the real interest rate , i_0 . This difference equation (6) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point i_0 the solution is highly sensitive to variations of the parameters b , c , g , and e ; secondly, given the parameters b, c, g , and e the solution is highly sensitive to variations of the initial point i_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

A key hypothesis of this work is based on the idea that the coefficient $\pi = b / (g+e)$ plays a crucial role in explaining local growth stability of the real interest rate, where, b - the coefficient of the investment function, g - the coefficient of the net capital outflow function, e - the coefficient of the saving function.

The estimated values of the coefficient π are smaller than 1 in the observed periods (0.742883; 0.570324; 0.997568). This model confirms decreasing real interest rate movement in USA in the observed periods.

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