The Chaotic Inflation Growth Model: The Euro Area

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Abstract

The basic aim of this analysis is to set up a relatively simple chaotic inflation growth model that is capable of generating stable equilibria, cycles, or chaos. It is important to analyze the local stability of inflation growth in the Euro Area in the period 2000-2013 (www.imf.org). A key hypothesis of this work is based on the idea that the coefficient $\mu = (1 - \rho)$ plays a crucial role in explaining local stability of inflation, where, ρ – the coefficient which explaines relation between unemployment rate growth rate and the fraction of the labor force that is not used up by the unemployment .This paper confirms continously fluctuations of inflation in the eurozone in the   period 2000-2013.

1   Introduction

The eurozone consists of the following EU Member States (18) which have adopted the euro as their single currency: Austria, Belgium, Cyprus , Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Luxemburg, Malta, Netherlands, Portugal, Slovakia, Slovenia, and Spain.
The inflation rate in the eurozone was at 0.80 percent in February of 2014. In the Euro Area, the inflation rate is calculated using the weighted average of the Harmonised Index of Consumer Price (HICP) aggregates. This index measures changes in the price level of consumer goods and services purchased by households. Inflation (average consumer prices, percent change) in the Euro area in the period 2000-2013 is presented in Figure 1.

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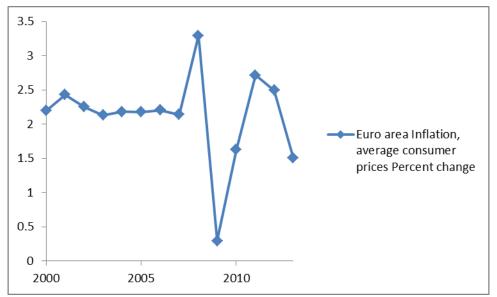
Figure 1: Inflation (average consumer prices, percent change) in   the Euro Area in the period 2000-2013. (www.imf.org)

Unemployment rate in the eurozone was at 12 percent in January of 2014.   Unemployment in the Euro Area was 19 million people in January of 2014. The unemployment rate measures the number of people actively looking for a job as a percentage of the labour force. Unemployment rate in the Euro area in the period 2000-2013, is presented in Figure 2.
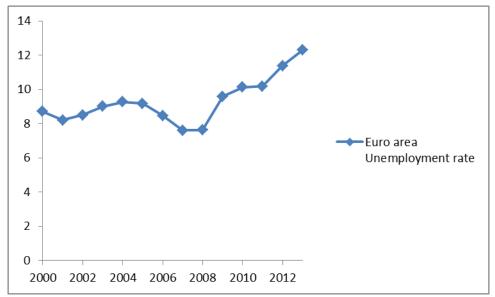

Figure 2: Unemployment rate in the Euro area in the period 2000-2013. (www.imf.org)

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local stability of inflation growth in the eurozone   in the period 2000-2013 (www.imf.org).

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983) , Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2011, 2012, 2013), Puu, T. (2003), Zhang W.B. (2006) , etc.

Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions ; (ii) cause and effect are not proportional; and (iii) nonlinearity.

2   The Model

The Phillips curve (1958) is a curve that shows the negative short-run tradeoff between inflation (Π) and unemployment (u) . To illustrate the short-run and long – run relationship between inflation and unemployment, Friedman (1968) and Phelps (1967) includes a new variable into the analysis : expected inflation (Π^e) . Expected inflation explains how much people expect the overall price level to change.

Thus, Friedman and Phelps concluded that:

$$u_t = u_n - \alpha \, (\Pi_t \quad - \Pi^e) \qquad (1)$$

Further, it is supposed:

$$u_n = \beta \, u_t \qquad (2)$$

$$\Pi^e \quad = \gamma \, \Pi_t \qquad (3)$$

Where : u_t – unemployment rate ; u_n – natural rate of unemployment; Π_t - actual inflation; Π^e – expected inflation ; α – the coefficient which explaines how much unemployment responds to unexpected inflation; β - the coefficient which relates unemployment rate and natural rate of unemployment; γ – the coefficient which explaines relation between actual and expected inflation;

Further, it is supposed that the growth rate of unemployment at time t should be proportional to (1 - u_t), the fraction of the labor force that is not used up by the unemployment at time t. Assuming that the unemployment is restricted by the labour force, the growth of the unemployment rate should change according th the following equation, after introducing a suitable parameter ρ (Jablanovic, 2011).

$$\frac{u_{t+1} - u_t}{u_t} = \rho \left(1 - u_t\right) \qquad (4)$$

Where : u_t – unemployment rate ; ρ – the coefficient which explaines relation between unemployment rate growth rate and the fraction of the labor force that is not used up by the unemployment at time t .

By substitution one derives:

$$\Pi_{t+1} = (1-\rho)\Pi_t - \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)} \right] \Pi_t^{\,2} \tag{5}$$

Further, it is assumed that the current value of inflation is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, inflation growth rate depends on the actual inflation, Π, relative to its maximal size in its time series Π^m. We introduce π as $\pi = \Pi/\Pi^m$. Thus π range between 0 and 1. Again we index π by t, i.e., write π_t to refer to the size at time steps t = 0,1,2,3,... Now inflation growth rate is measured as

$$\pi_{t+1} = (1-\rho)\pi_t - \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)} \right] \pi_t^{\,2} \tag{6}$$

This model given by equation (6) is called the logistic model. For most choices of α , β , γ , and ρ there is no explicit solution for (6). Namely, knowing α , β , γ , ρ , and measuring π_0 would not suffice to predict π_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (6) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of π_t. This difference equation (6) will posses a chaotic region. Two properties of the chaotic solution are important : firstly, given a starting point π_0 the solution is highly sensitive to variations of the parameters α , β , γ , and ρ ; secondly, given the parameters α , β , γ , and ρ , the solution is highly sensitive to variations of the initial point π_0 . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

3   The Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \mu z_t (1 - z_t) \; , \qquad \mu \in [\,0,4\,] \qquad , \qquad z_t \in [\,0,1\,] \tag{7}$$

is equivalent to the iteration of growth model (6) when we use the identification

$$z_t = \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)(1-\rho)}\right]\pi_t \qquad \text{and} \qquad \mu = (1-\rho) \qquad\qquad (8)$$

Using (6) and (8) we obtain:

$$z_{t+1} = \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)(1-\rho)}\right]\pi_{t+1} =$$

$$= \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)(1-\rho)}\right]\left\{(1-\rho)\pi_t - \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)}\right]\pi_t^{\,2}\right\}$$

$$= \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)}\right]\pi_t - \left[\frac{\rho^2\,\alpha^2\,(\gamma-1)^2}{(1-\beta)^2\,(1-\rho)}\right]\pi_t^{\,2}$$

On the other hand, using (7) and (8) we obtain:

$$z_{t+1} = \mu\,z_t\,(1-z_t) =$$

$$= (1-\rho)\left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)(1-\rho)}\right]\pi_t\left\{1-\left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)(1-\rho)}\right]\pi_t\right\} =$$

$$= \left[\frac{\rho\,\alpha\,(\gamma-1)}{(1-\beta)}\right]\pi_t - \left[\frac{\rho^2\,\alpha^2\,(\gamma-1)^2}{(1-\beta)^2\,(1-\rho)}\right]\pi_t^{\,2}$$

Thus we have that iterating (6) is really the same as iterating (7) using (8) (see Fig.3.) . It is important because the dynamic properties of the logistic equation (7) have been widely analyzed (Li and Yorke (1975), May (1976)).
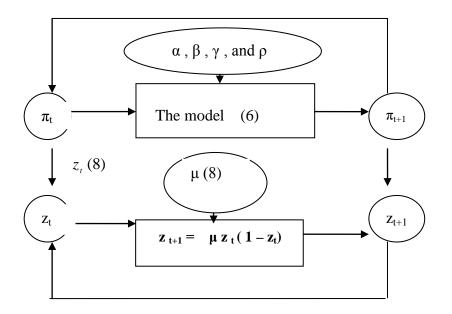
Figure 3: Two quadratic iteratiors running in phase are tightly coupled by the transformations indicated

It is obtained that :

(i) For parameter values $0 < \mu < 1$ all solutions will converge to $z = 0$;

(ii) For $1 < \mu < 3,57$ there exist fixed points the number of which depends on μ;

(iii) For $1 < \mu < 2$ all solutions monotnically increase to $z = (\mu - 1) / \mu$;

(iv) For $2 < \mu < 3$ fluctuations will converge to $z = (\mu - 1) / \mu$;

(v) For $3 < \mu < 4$ all solutions will continously fluctuate;

(vi) For $3,57 < \mu < 4$ the solution become "chaotic" wihch means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

4   Empirical Evidence

The main aim of this paper is to analyze the inflation growth stability in the Euro Area in the period 2000-2013 , by using the presented non-linear, logistic inflaltion growth model (9) :

$$\pi_{t+1} = \mu \pi_t - \vartheta \pi_t^2 \tag{9}$$

where : π – inflation , $\mu = (1 - \rho)$, $\vartheta = [\rho \alpha (\gamma - 1)] / (1 - \beta)$.

Firstly, data on inflation are transformed (www.imf.org) from 0 to 1, according to our supposition that actual value of inflation , Π , is restricted by its highest value in the time-series, Π^m. Further , we obtain time-series of $\pi = \Pi / \Pi^m$.

Table 1: The estimated model (9): Euro Area , 2000-2013. (R= 0.75114 Variance explained: 56.421%) (www.imf.org)

	μ	ϑ
Estimate	3.113266	3.010611
Std.Err.	0.337471	0.446286
t(29)	9.225293	6.745923
p-level	0.00000	0.000032

5   Conclusion

This paper suggests conclusion for the use of the chaotic inflation growth model in predicting the fluctuations of inflation.   The model (6) has to rely on specified parameters α , β , γ , and ρ , and initial value of inflation , π_0. But even slight deviations from the values of parameters: α , β , γ , and ρ and initial value of inflation , π_0 show the difficulty of predicting a long-term inflation.

A key hypothesis of this work is based on the idea that the coefficient $\mu = (1 - \rho)$ plays a crucial role in explaining local stability of inflation, where, ρ – the coefficient which explaines relation between unemployment rate growth rate and the fraction of the labor force that is not used up by the unemployment at time t.

The estimated value of the coefficient μ is 3.113266. This result confirms continously fluctuations of inflation in Euro Area in the   period 2000-2013. Decreasing unemployment rate is important ingredient of inflation growth rate stability in Euro Area.

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