Advances in Management & Applied Economics, vol. 3, no.5, 2013, 21-39
ISSN: 1792-7544 (print version), 1792-7552(online)
Scienpress Ltd, 2013

Evaluating Staff Performance: A Markov Chain Approach

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Abstract

The purpose of this paper is to apply and validate an application of Markov chain models to measure the effects of different staffing levels on group performance whilst including the effects of absenteeism. Two models were formulated, one that models absenteeism in detail, and another that uses a simplified approach. Experiments and Monte Carlo simulations were conducted to confirm the validity of the models. Both Markov chain models provide results that fit within the standard error of the experiments. Limitations of this research are: (1) only one piece of work arrives at a time; (2) the arrival rate of work is a constant; (3) work needs to be executed on arrival, otherwise it leaves the system immediately and the group loses that piece of work; and (4) only one worker begins an absence at any time. This research adds to the literature on organisation management in two distinct ways. First, it shows that a Markov chain model is able to accurately include the effects of absenteeism on the relationship between staffing levels and performance of a group. Second, the Markov chain model can be simplified without loss of accuracy.

JEL classification numbers: C30, C32, C61, L10, L20, M51, M54.
Keywords: Markov chain, Staffing level, Staffing optimisation, Organisation performance, Absenteeism.

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Article Info: *Received :* April 16, 2013. *Revised :* May 28, 2013.
Published online : September 1, 2013

1 Introduction

Absenteeism is a common problem for work groups. Workers may be absent for a variety of reasons including illness and vacations. If one or more workers are absent then the capacity of the workgroup for carrying out work is reduced. This needs to be considered in the initial sizing of the work group or the work group will be unable to carry out its allocated work as expected.

The aim of this study is to apply and validate an application of a Markov chain model to measure the effects of different staffing levels on group performance in the presence of absenteeism. Thus a worker in this model is available to carry out work only if they are neither absent, nor currently engaged in previous work. The staff in the group are assumed to be organised as a parallel server system and work individually. Other assumptions for the study include: (1) only one piece of work arrives at a time; (2) the arrival rate of work is a constant; (3) work needs to be executed on arrival, otherwise it leaves the system immediately and the group loses that piece of work; and (4) only one worker commences leave at any time (although multiple workers may be continuing leave from previous periods).

The term 'performance' in the literature has a broad range of meanings, including financial performance, business performance, strategic performance, and so on [1]. Group performance in this research is limited to three indicators: (1) the group's probability of carrying out work as it arrives, (2) the number of available staff, and (3) the group utilisation.

These three indicators were also used in Tran and Davis [2] to model the relationship between staffing level and performance of a work group. However, in the previous study the only reason why workers were considered to be unavailable to accept arriving work was because they are already processing work, and no other reasons why they may be unavailable were considered. The present study extends the model to include absenteeism by workers in an organisation. Two different approaches are used. The first approach models leave in detail, while the second approach uses a simplified method. Experiments and Monte Carlo simulations were conducted to validate the theoretical model. Comparisons between the two methods are also made to see which approach is better in terms of accuracy in modelling and tractability in calculation.

This paper is organised as follows. Section 2 reviews the literature on the effects of staffing levels on performance, the quantitative approach in managing human resources, and some applications of Markov chains in the literature. Section 3 presents the modelling process using a fully detailed approach that uses the full transition matrix including both absent states and occupied states; whereas an approach using a simplified transition matrix is presented in Section 4. Section 5 describes the experiments; while the Monte Carlo simulation process is given in Section 6. Section 7 compares the results of the experiments and the simulations with the two Markov chain approaches. Conclusions of the study are made in the last section.

2 Background

2.1 The Effects of Staffing Levels on Organisation Performance

Workforce sizes have different effects on performance of a group. However, the literature has not reached an agreement on the relationship between levels of staff and group's performance. Researchers have approached the issues from different views and hence come up with different results. On one hand, some scholars conclude that a group should be at a moderate understaffing level as this state improves organisational performance. On the other hand, others prove that a slight overstaffing condition has a positive relationship with performance. There is, however, common agreement that both great overstaffing and extreme understaffing conditions have negative effects on the performance of an organisation.

An understaffing condition is defined as "not having enough people to do all the jobs in the setting" [3]. A moderately understaffed organisation may gain better performance compared with other conditions since employees in this state are likely to work more efficiently and to experience higher motivation. In a slightly understaffed state, each member is involved in a wider variety of tasks. Hence they need to use varied skills to complete those tasks. Furthermore, workers also need to find ways to combine similar tasks together in order to reduce wasted time from repeating redundant activities. As a consequence, their efficiency is improved [4, 5]. Workers in understaffed conditions are also claimed to have more freedom on deciding the way to organise and complete their tasks which increases their working motivation [6]. These positive experiences lead to improvement in the outputs of individuals and hence the performance of the whole organisation is improved.

However, negative effects on workers have been reported in understaffed organisations. Studies show that workers in understaffed conditions normally suffer from higher burnout and higher emotional exhaustion [7]. This leads to demotivation, lower productivity and poor performance among individuals [8]. A high absenteeism rate is another effect of understaffing conditions. Psychologists indicate that high staff burnout, less job satisfaction, high sickness rate, and high conflict in work–life balance are the main reasons for absenteeism by workers [9-11]. Other disadvantages of understaffing claimed by researchers include lower levels of aggregate organisation's outputs, lost business opportunities, and an increase in error rates of staff [12].

To overcome the harmful effects that understaffing conditions have on performance, other researchers suggest a slight overstaffed setting for an organisation. Numerous studies have shown that this condition is associated with better outcomes at both staff and organisation levels. Rafferty, et al. [8], for example, reports that employees in overstaffed groups suffer less from burnout, have higher job satisfaction, and produce a higher quality of service. At a strategic level, the existence of extra staff enables creative and innovative behaviours [13], and enhances the performance of the organisation [14]. Researchers acknowledge that increasing the staffing level leads to higher costs. Hence they suggest that an organisation should not be greatly overstaffed [15, 16]. After confirming the existence of an optimal level of additional staff, Tan [15] concludes that the resource of extra staff can be a source of competitive advantage but that too many staff may reduce organisation performance.

2.2 The Quantitative Approach in Setting Staffing Levels

Most of the foregoing research was qualitative in nature. In addition quantitative methods have also been applied to this problem. Che and Henderson [17], for example, apply queuing models in setting levels of tellers for call centres. The work addresses errors of prior studies when using the theory to the problem and proposes a priority queuing model for identifying the number of call takers for different periods to enable the centre meets customer demands. In addition, queuing models are also applied in setting the number of nurses for hospitals [see e.g. 18], or in optimising the staffing level based on profit [19, 20]. Other mathematical models are used for monitoring training costs and times, maximising the flexibility of the workforce, and optimising the trade–off between the cost of training and the flexibility of employees [see e.g. 21, 22, 23].

2.3 Applications of Markov chain models

Markov chain models have been applied in diverse areas such as wireless communication, financial engineering, internet traffic modelling and so on [24, 25]. The models have also been applied in the construction industry [see e.g. 26] and in project management [see e.g. 27]. In the field of human resource management, Markov models are used in allocating employees to different parts of a firm [28], or the changes of workforce structure in an organisation [29].

One such application of Markov chain models in modelling organisation performance is Tran and Davis [2]. The work models the relationship between staffing levels and performance of a group. However, it considers workers to be unavailable only because they are serving previously arrived work, and does not consider other reasons why they may be unavailable. The present study extends the model to one such case, namely the case where workers are unavailable due to absenteeism. The model provides a practical tool for managers in evaluating the performance of their work groups. The tool is also useable for those who want to setup an appropriate staffing level for a newly formed team.

3 Modelling – The Combined Approach

The combined approach presented in this section seeks to accurately keep track of workers that are occupied and workers that are on leave. These two states of workers are independent of each other so they need to be modelled separately before being joined to make a combined model. The modelling processes for the combination are as follows.

3.1 Symbols Used

i	Number of occupied workers before a given time step
i'	Number of workers on leave before a given time step
j	Number of occupied workers after a given time step
j'	Number of workers on leave after a given time step
k	State of the whole system ($0 \le k \le m$), $k = j + j'$
m	Number of workers in the group
n	Maximum number of workers on leave at a particular time ($n \le m$)
p	Probability of finishing work of a worker during a time step
P	Transition matrix of the whole system
P'	Transition matrix of the on leave worker system
P_o	Transition matrix of the occupied workers system

P_f' Transition matrix of the system where workers finish an absence
P_s' Transition matrix of the system where workers start an absence
$p_{(f)i'j'}$ Transition probability of the on leave system where i' workers finish on leave to j' workers finish on leave
$p_{(s)i'j'}$ Transition probability of the absent system where i' workers start on leave to j' workers start on leave
p_f Probability of a worker finishing an absence
$p_{i'j'}$ Transition probability of the absent system from the state of i' workers are on leave to the state of j' workers are on leave
$p_{ii'jj'}$ Transition probability of the whole system from the state of i occupied workers and j absent workers to the state of i' occupied workers and j' absent workers
p_{ij} Transition probability of the occupied worker system from the state of i workers are occupied to the state of j workers are occupied
p_s Probability of a worker starting an absence
λ The average arrival rate of work, hence $1/\lambda$ is the inter–arrival time
μ The average service rate, hence $1/\mu$ is the average time for doing one piece of work of a worker
μ' The average away rate, hence $1/\mu'$ is the average time for being on leave of a worker
π_j Probability of the whole system being in state of j occupied workers
$\pi_{j'}$ Probability of the whole system being in state of j' absent workers
π_k Probability of the whole system being in state of k unavailable workers

3.2 Modelling the Occupied Workers

The occupied workers are those processing work when a piece of work arrives. The model used here is very similar to that presented in Tran and Davis [2].
The transition matrix of the occupied workers is given by:

$$P_o = [p_{ij}] = \begin{bmatrix} p_{00} & p_{01} & \cdots & p_{0m} \\ p_{10} & p_{11} & \cdots & p_{1m} \\ & \cdots\ldots & & \\ p_{m0} & p_{m1} & \cdots & p_{mm} \end{bmatrix} \quad (1)$$

Where

$$0 \le p_{ij} \le 1 \quad (2)$$

and

$$\sum_{j=0}^{m} p_{ij} = 1 \qquad (i = 0, \ldots, m) \quad (3)$$

If there were i workers occupied in the previous time step then the transition probability of the system, p_{ij}, represents the probability of (i – j + 1) workers finishing their work during the time step. The +1 results from the new work arriving. All workers being occupied and none finishing represents a special case dealt with below. The probability for a given number of workers to finish their work during a time step can be modelled by a binomial distribution [2]. There are two cases for starting a work when it arrives. Either there is a worker available to start the work or there is not.

- In the case where there is a worker available (i.e. when $i < m$) then p_{ij} is given by:

$$p_{ij} = \binom{i+1}{j} p^j (1-p)^{i+1-j} \quad (4)$$

Where p is the probability of a worker finishing his or her work during a time step.

- When $i = m$ then $p_{ij} = p_{(i-1)j}$. This is because when the system is either in state (m–1) or in state m all workers are occupied after a piece of work arrives. In the former case the only available worker begins to carry out the new piece of work. In the latter case all workers are already occupied and the new piece of work is not carried out.

It is assumed that the time taken to carry out each piece of work can be described using an exponential distribution. Hence, the probability that a worker finishes his or her work during a time step is given by:

$$p = \exp(-\mu/\lambda) \quad (5)$$

The row vector indicating the probability of being in a state of the system is:

$$\pi^o = (\pi_0 \ \ \pi_1 \ \ldots \ \pi_j \ \ldots \ \pi_m) \quad (6)$$

where

$$0 \leq \pi_j \leq 1 \quad (7)$$

and

$$\sum_{j=0}^{m} \pi_j = 1 \quad (8)$$

If the system is in equilibrium then the probabilities of being in any state do not change between time steps and:

$$\pi^o = \pi^o P_o \quad (9)$$

Equations (8) and (9) give (m+2) equations with (m+1) unknowns. However, the row vectors comprising P_o in Equation (9) are linearly dependent and so one of the equations in Equation (9) will be eliminated. This gives (m+1) equations in (m+1) unknowns. Solving this gives the probability of being in each state, π_j.

3.3 Modelling the Absent Workers

It is assumed that there is always a fixed probability of exactly one worker beginning an absence (unless all workers are already absent). The probability of a worker finishing an absence in a given time step is assumed to follow a binomial distribution. The transition matrix of the absences is given by:

$$P' = \left[p_{i'j'}\right] = p_s * p_{(f)(i'+1)j'} + (1 - p_s) * p_{(f)i'j'} \quad (10)$$

Where:

- p_s is the probability that any worker will start an absence this time step
- $p_{(f)(i')j'}$ is the probability that (i' – j') workers will return from an absence if (i') workers were absent at the beginning of the time step.

- When $i' \leq n$ then:

$$p_{(f)i'j'} = \binom{i'+1}{j'} p_f^{j} (1-p_f)^{i'+1-j'} \quad (11)$$

- When $i' > n$ then $p_{(f)i'j'} = 0$
- When $i' + 1 = n$ then $p_{(f)(i'+1)j'} = p_{(f)i'j'}$. This only applies when n<m and there is an absent worker coming back at the same time that a different worker starts an absence.

The row vector indicating the probability of being in a state of the system in this case is:

$$\pi' = (\pi_0 \ \ \pi_1 \ \ldots \ \pi_{j'} \ \ldots \ \pi_n) \quad (12)$$

Where

$$0 \leq \pi_{j'} \leq 1 \quad (13)$$

and

$$\sum_{j'=0}^{n} \pi_{j'} = 1 \quad (14)$$

When the system is in equilibrium then:

$$\pi' = \pi' P' \quad (15)$$

Since equations (14) and (15) have (n + 2) equations with (n + 1) unknowns, eliminating one equation in Equation (15) and solving Equations (14) and (15) will give the probability of being in each state, $\pi_{j'}$.
The final results from the modelling processes of the occupied workers and the absent workers (π_j and $\pi_{j'}$) are used to calculate the group utilisation that will be presented in Section 3.e below. The intermediate results (P_o and P') are used for the following section.

3.4 Modelling the Whole System

The state of the system is defined by two dimensions, jj'. The transition matrix has [(m+1)*(m+2)–(m–n)*(m–n+1)]/2 x [(m+1)*(m+2)–(m–n)*(m–n+1)]/2 dimensions and is given by:

$$P = [p_{ii\ jj'}] = \begin{bmatrix}
p_{00\ 00} & p_{00\ 10} & \cdots & p_{00\ m0} & p_{00\ 01} & p_{00\ 11} & \cdots & p_{00\ (m-1)1} & \cdots & p_{00\ 0n} & p_{00\ 1n} & \cdots & p_{00\ (m-n)n} \\
p_{10\ 00} & p_{10\ 10} & \cdots & p_{10\ m0} & p_{10\ 01} & p_{10\ 11} & \cdots & p_{10\ (m-1)1} & \cdots & p_{10\ 0n} & p_{10\ 1n} & \cdots & p_{10\ (m-n)n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p_{m0\ 00} & p_{m0\ 10} & \cdots & p_{m0\ m0} & p_{m0\ 01} & p_{m0\ 11} & \cdots & p_{m0\ (m-1)1} & \cdots & p_{m0\ 0n} & p_{m0\ 1n} & \cdots & p_{m0\ (m-n)n} \\
p_{01\ 00} & p_{01\ 10} & \cdots & p_{01\ m0} & p_{01\ 01} & p_{01\ 11} & \cdots & p_{01\ (m-1)1} & \cdots & p_{01\ 0n} & p_{01\ 1n} & \cdots & p_{01\ (m-n)n} \\
p_{11\ 00} & p_{11\ 10} & \cdots & p_{11\ m0} & p_{11\ 01} & p_{11\ 11} & \cdots & p_{11\ (m-1)1} & \cdots & p_{11\ 0n} & p_{11\ 1n} & \cdots & p_{11\ (m-n)n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p_{(m-1)1\ 00} & p_{(m-1)1\ 10} & \cdots & p_{(m-1)1\ m0} & p_{(m-1)1\ 01} & p_{(m-1)1\ 11} & \cdots & p_{(m-1)1\ (m-1)1} & \cdots & p_{(m-1)1\ 0n} & p_{(m-1)1\ 1n} & \cdots & p_{(m-1)1\ (m-n)n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p_{0n\ 00} & p_{0n\ 10} & \cdots & p_{0n\ m0} & p_{0n\ 01} & p_{0n\ 11} & \cdots & p_{0n\ (m-1)1} & \cdots & p_{0n\ 0n} & p_{0n\ 1n} & \cdots & p_{0n\ (m-n)n} \\
p_{1n\ 00} & p_{1n\ 10} & \cdots & p_{1n\ m0} & p_{1n\ 01} & p_{1n\ 11} & \cdots & p_{1n\ (m-1)1} & \cdots & p_{1n\ 0n} & p_{1n\ 1n} & \cdots & p_{1n\ (m-n)n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
p_{(m-n)n\ 00} & p_{(m-n)n\ 10} & \cdots & p_{(m-n)n\ m0} & p_{(m-n)n\ 01} & p_{(m-n)n\ 11} & \cdots & p_{(m-n)n\ (m-1)1} & \cdots & p_{(m-n)n\ 0n} & p_{(m-n)n\ 1n} & \cdots & p_{(m-n)n\ (m-n)n}
\end{bmatrix} \quad (16)$$

Where $p_{ii'\,jj'}$ is the transition probability of the system from the state of i occupied workers and i' absent workers to the state of j occupied workers and j' absent workers and is given as follows:

- For i ≥ (m – j') (i.e. when the number of occupied workers at the beginning of the time step is greater than the number of workers who are not absent at the end of the time period, i.e. no one is available to accept incoming work), then

$$p_{ii'\,jj'} = p_{(m-j'-1)i'\,jj'} \qquad (17)$$

- For all other cases:

$$p_{ii'\,jj'} = p_{ij} \times p_{i'j'} \qquad (18)$$

This retains the important properties that:

$$0 \leq p_{ii'\,jj'} \leq 1 \qquad (19)$$

and

$$\sum_{j'=0}^{n} \sum_{j=0}^{m-j'} p_{ii'\,jj'} = 1 \qquad (i' = 0, \ldots, n;\ i = 0, \ldots, m - i') \qquad (20)$$

A simplified system state, k, is defined as the number of unavailable workers, including the number of occupied workers and the number of absent workers, $k = j + j'$, $0 \leq k \leq m$. The probability of being in state k is the probability of the group having exactly k workers unavailable, π_k, and given by:

$$\pi_k = \sum_{j'=0}^{\min(k,n)} \pi_{(k-j')j'} \qquad (k = 0, \ldots, m) \qquad (21)$$

The group is in the "not available" state when all workers are unavailable. All workers in this state are either on leave or occupied. If work arrives at this time then no workers are available to immediately commence the arriving work. The probability of being in this state is π_m. Thus the probability of accepting a piece of work is the probability that any worker is available, 1- π_m.
The row vector indicating the probability of being in a state of the system is:

$$\pi = (\pi_{00}\ \ \pi_{10} \ldots \pi_{jj'} \ldots \pi_{(m-n)n}) \qquad (22)$$

where

$$0 \leq \pi_{jj'} \leq 1 \qquad (23)$$

and

$$\sum_{j'=0}^{n} \sum_{j=0}^{m-j'} \pi_{jj'} = 1 \qquad (24)$$

When the system is in equilibrium then:

$$\pi = \pi P \quad (25)$$

Equations (24) and (25) give [(m+1)*(m+2)–(m–n)*(m–n+1)]/2 equations with [(m+1)*(m+2)–(m–n)*(m–n+1)]/2 – 1 unknowns. However, one of the equations in Equation (25) is eliminated since the row vectors comprising P are linearly dependent. This gives [(m+1)*(m+2)–(m–n)*(m–n+1)]/2 equations in [(m+1)*(m+2)–(m–n)*(m–n+1)]/2 unknowns. Solving this gives $\pi_{jj'}$. Equation (21) can be used to get the probability of being in each state, π_k.

3.5 Organisation Utilisation

The utilisation of an organisation is the ratio between the average number of occupied staff to the average number of workers at the workplace. It is given by:

$$H = N_{\text{occupied staff}} / N_{\text{workers at the workplace}} \quad (26)$$

Where

$$N_{\text{occupied staff}} = \sum_{j=0}^{m} \pi_j j \quad (27)$$

And

$$N_{\text{workers at the work place}} = m - \sum_{j'=0}^{n} \pi_{j'} j' \quad (28)$$

4 Modelling – A Simplified Approach

The combined approach presented in Section 3 quickly becomes intractable as the number of employees and/or potential absentees increases. Therefore a simplified approach to the problem with a much smaller transition matrix is proposed in this section. The validity of the approach will be confirmed by comparing the results from the simplified model with the experimental results and the simulation results in later sections. The results from the approach are also compared with the results from the method presented in the previous section to determine how much accuracy is lost in the simplified process.

The two independent sub-systems modelled in Section 3 will become one system if the absent workers are considered as ‘occupied’ workers. This means that the number of absent workers is treated as being occupied but they do not produce outputs for the group. When the probability of being absent is known, then the average service time of the group in the simplified approach is given by:

$$\mu_{sa} = (1 - p_l)\mu + p_l\mu' \quad (29)$$

Where p_l is the probability that a worker is absent during a time step.

Using μ_{sa} instead of μ for Equation (5) enables the simplified transition matrix of the system to be established through Equations (1) to (4). Solving Equations (8) and (9) will return the probability of being in any state of the whole system, π_j, for the simplified method. While this method does not distinguish between occupied and absent workers, it does furnish the number of workers that are available.

5 Experiments

5.1 Experimental Design

Experiments were conducted on work groups where workers independently disassembled electronic components from circuit boards. The number of workers in the experimental groups ranged from 8 to 12 and they were organised as parallel systems.

Work arrived at a constant rate and one piece at a time. Each piece of incoming work was allocated to a single randomly selected worker. In the case that no workers were available to accept new work at the time that a piece of work arrived, then the piece of work left the system and was unable to be executed by the group.

Workers were absent randomly during the experiments. In order to simulate the fact that absences tend to be a whole number of days each experiment was broken up into intervals consisting of 30 time steps each (some experiments used intervals consisting of 15 time steps). A maximum of one worker was allowed to begin leave at any particular time and if this occurred it would take place at the beginning of the interval. The maximum time that a worker was away from the workplace was 150 time steps. The time for starting and finishing each absence was a multiple of the interval length.

5.2 Data Collection

Data collected from the experiments included arrival times, service times, and finishing times of each piece of work. For each period of leave the starting time and the finishing time were recorded. Each experiment lasted for 1440 time steps. After being stopped the experiments were repeated a second time from the beginning.

Four set of experiments with differences in (1) the probabilities of an absence beginning, (2) the number of time steps between potential absences beginning, (3) and the durations for leave were conducted. The following table summarizes the conditions of all the sets of experiments.

Table 1: Conditions of the experiments

The experiment set	**Probability of away workers**	**Time for starting leave**	**Duration for leave**
1st	40%	A multiple of 30 time steps	A multiple of 30 time steps
2nd	20%	A multiple of 30 time steps	A multiple of 30 time steps
3rd	20%	A multiple of 15 time steps	A multiple of 30 time steps
4th	20%	A multiple of 15 time steps	A multiple of 15 time steps

5.3 Simulation Experiments

The results from the experiments did not line up perfectly with the theoretical model and so Monte Carlo simulations were carried out to see if this was because the assumptions in the model did not perfectly fit the probability distributions in the experiments, or if the experiments were too short. For each group's simulation, the number of workers, the average service time, the probability of a worker starting an absence, the probability of a worker finishing an absence, and the maximum number of concurrent absences were matched to the experiment. Service times of workers were assumed to follow an exponential distribution, whereas the number of workers returning after an absence each time step was assumed to follow a binomial distribution. A 10,000 time step simulation was run 30 times for each group. Data taken from the simulation for each group includes the probability distribution of unavailable workers, the probability distribution of occupied workers, and the probability distribution of absences.

6 Comparisons of Experimental Results and Simulation Results with the Two Approaches

Results from the experiments as well as from the simulation processes are compared with the two approaches of the theoretical model in the three aspects of performance mentioned in Section 1. Discussions on the two approaches are also made. The comparisons are as follows.

6.1 Probability of Carrying out Work

The study assumes that all work being accepted by groups will be processed successfully. So the probability of carrying out work here is the probability of accepting work. As mentioned in previous sections, the probability of a group accepting a piece of work is the probability that at least one worker is available when a piece of work arrives. The model gives this as $1 - \pi_m$. For the experiments this is taken to be the ratio between the amount of work carried out to the amount of work arriving. In the simulations, the probability of carrying out work is $1 - p_m$ where p_m is the probability that m workers are unavailable.

Similar to Tran and Davis (2011; 2012) increasing the number of workers in a group leads to an increased probability of accepting work, and adding an additional worker to a small group has a bigger effect than adding to an already large group. For example, the probability of carrying out work increased about 1.5%, from about 98% to 99.5%, when adding one extra worker to the 8–worker group. However, probability of carrying out work was almost unchanged when the group enlarged from 11 to 12 workers (Figure 1.a).

Figure 1 illustrates the probabilities of accepting arrival work for all experiments. Results from the experiments and the simulations are compared with the results from the two theoretical models presented in Section 3 and Section 4. Error bars give the standard error for the mean of the experimental results. In order to determine these error bars, each experiment was divided into intervals of 150 seconds. Work arrived at a rate of one piece of work every five seconds so 30 pieces of work would arrive during each 150 second interval. The number of pieces of lost work for each interval was counted. The figure indicates that the theoretical results of the two methods are consistently within the standard error for all experimental groups, except the only one result from the simplified method at the 8–worker group in the 4^{th} experiment (Figure 1.d). As the error bars are expected to cover 95% of the probability space it is acceptable for 1 of the 20 data points to lie outside. This implies that both theoretical approaches proposed in this study provide reliable methods to model the relationship between workforce size and probability of carrying out work.

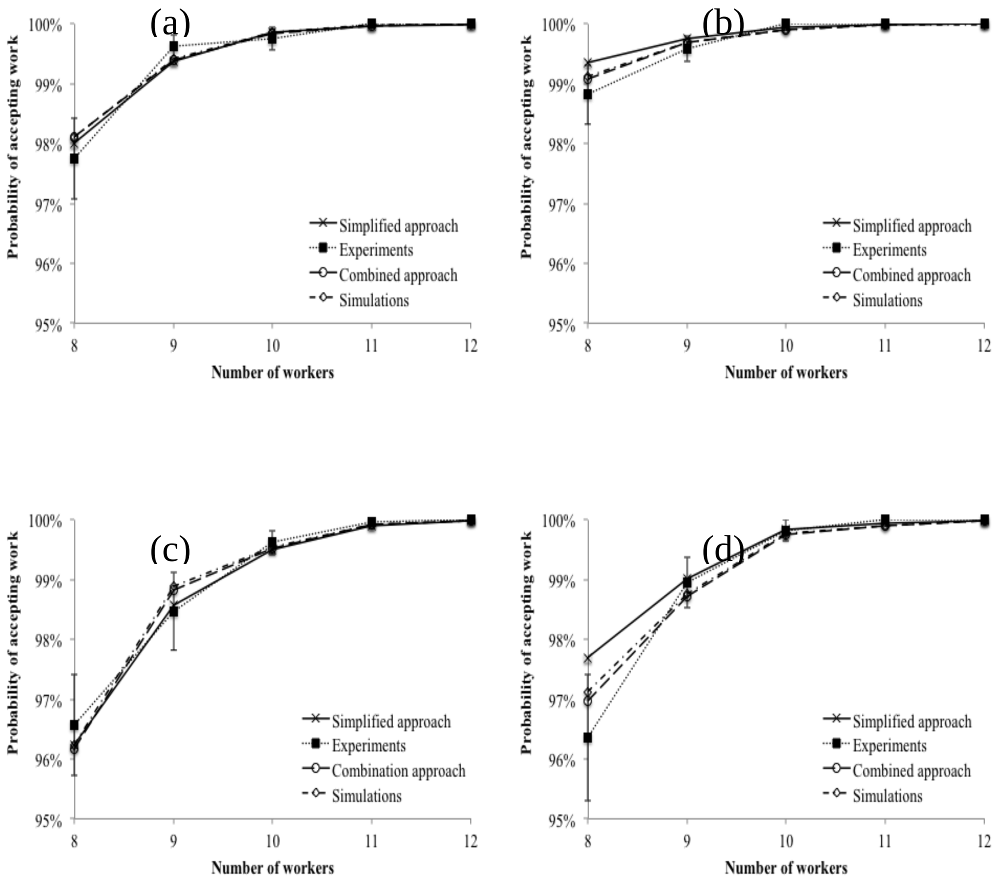

Figure 1: Probabilities of accepting work by groups in the 1st (a), 2nd (b), 3rd (c), and 4th (d) experiments

6.2 Number of Available Workers

Figure 2 illustrates the probability distribution of the number of available workers of the 8–worker group in each experiment and comparisons with the simulation and both theoretical approaches. Error bars are provided to show the standard error of the experimental results. shows that both theoretical curves fall in between error bars in all experiments. Kolmogorov – Smirnov goodness of fit test was also carried out for each different sized group of workers. Results from the tests and the figures indicate that the theoretical results from the two approaches fit the experimental results well for each experimental group in all experiments. Similar results were also found for other group sizes.

All the results from theoretical models, the simulations, and the experiments imply that the probability distribution of available workers has an inverted U-shape. For the 8–worker group in this study, the probability that four or more workers were available in the experiments was about 50%. This would appear to indicate that the group is overstaffed since the percentage of idle workers was high. Reducing the number of workers in the group may be beneficial. However, this will have the consequence of reducing the

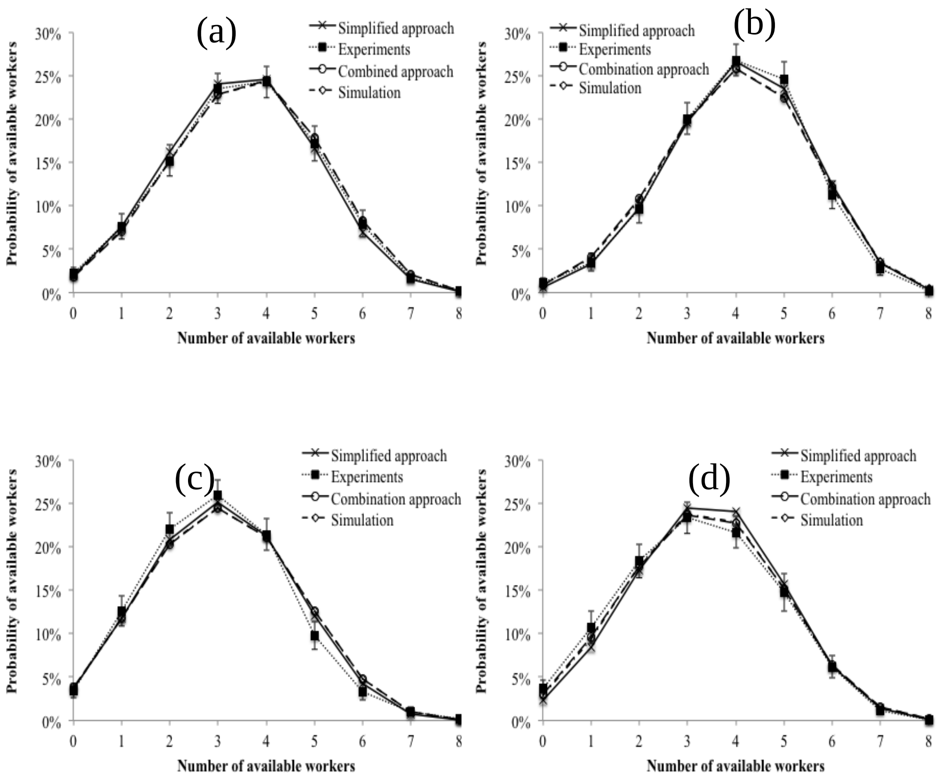

Figure 2: Probability distributions of available workers in the 8–worker group in the 1st (a), 2nd (b), 3rd (c), and 4th (d) experiments

total amount of work accepted. The optimum number of workers for the group may be identified by solving the trade–off problem between the costs of excess workers and the income from the extra work that those workers allow.

6.3 Group Utilisation

The utilisation for each group size is also examined in this study. Group utilisation was defined in Section 3 as “the ratio between the average number of occupied staff to the average number of staff at the workplace”. The indicator is also the ratio between the average actual outputs of the group to the maximum that could be produced per unit of time with the existing number of workers.

The group utilisation can be calculated by using Equations (8) and (9). For the experiments and the simulations, the average number of occupied workers and the probability of absences are identified; hence the group utilisation is easily calculated. Of the two theoretical methods, only the combined approach can be used to calculate the utilisation of a group since it specifies who is on leave and who is working. The simplified approach, on the other hand, does not make this distinction and so it cannot calculate the utilisation.

The probabilities of being absent and of being occupied can be determined by summing the appropriate $\pi_{jj'}$ from Equation (15). These then are put into Equations (8) and (9) for the theoretical results of the group utilisations.

Figure 3 presents the results from the experiments, the simulations, and the theoretical model. All the results indicate that when the arrival rate of work is unchanged, the utilisation of a group decreases when increasing the number of workers. This can be explained because although increasing the number of workers increases the probability of accepting work (as seen in Figure 1) the additional work accepted for each additional worker decreases as the number of workers increases. Eventually with a very large number of workers there is a negligible amount of extra work accepted and effectively all that happens is that the current workload of the group is shared by a larger number of workers. Hence, the workload for each worker decreases and each worker spends more time in the idle state.

Figure 3 shows that the model results are consistently within the standard error of the experimental results and very close to the simulation results. This shows that the theoretical model fits the experiments and the simulations well. However, the model and the simulation do tend to overestimate the experimental results. This is because the probability of absence is applied at different levels. The probability of absent workers is applied once per interval in the experiment. On the other hand, in the simulations and in the theoretical model, this probability is divided by 30, which is the number of time steps in an interval, but then applied at every time step.

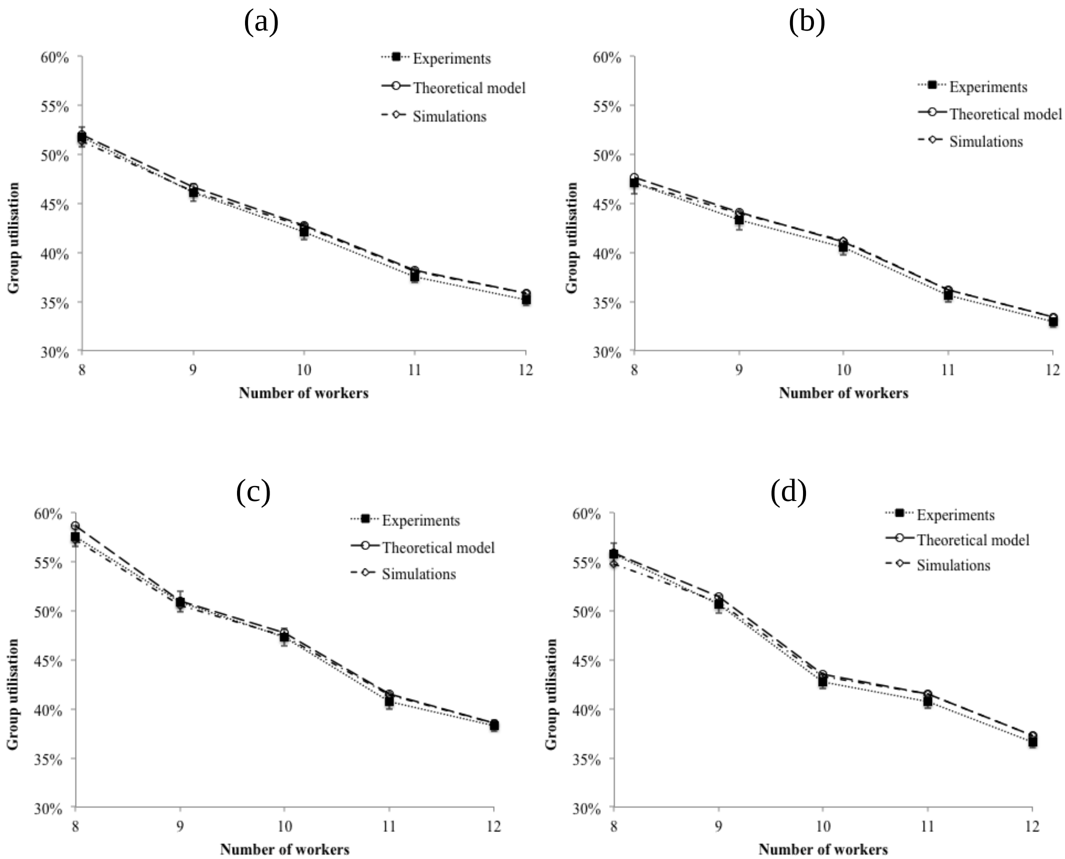

Figure 3: Utilisations of groups in the 1st (a), 2nd (b), 3rd (c) and 4th (d) experiments

6.4 Comparison of the Two Approaches

All figures in above sections indicate that both of the theoretical approaches provide reliable tools for modelling the relationship between staffing level and performance of a group. All theoretical results are consistently within the standard errors of the experimental results.

The combined method gives all details of the system, including the probability of absences, the probability of occupied workers and the utilisation of a group. However, the method quickly becomes intractable when the total number of workers, m, and the number of workers potentially absent, n, of a group increase. This is because the method has a transition matrix with $[(m+1)*(m+2)–(m–n)*(m–n+1)]/2$ x $[(m+1)*(m+2)–(m–n)*(m–n+1)]/2$ dimensions.

On the other hand, the simplified approach uses a transition matrix with only (m+1) x (m+1) dimensions and the results are still within the standard errors. However, the approach does not provide some of the details that the combined method does such as the probability of absences, the probability distribution of occupied workers, and the utilisation of the examined group.

7 Conclusions

The study proves that the Markov chain model provides a practical and useful tool to model the relationship between staffing level and performance of a group. The models presented in this paper allow the inclusion of the case where workers may be unavailable to accept arriving work due to being absent. Two different approaches have been validated. Results from this study indicate that the simplified method is easier in calculation while still maintaining accuracy compared to the combined method. The combined approach, on the other hand, provides more details of the investigated groups. A practitioner may make a decision on choosing which method is most suitable for modelling the relationship between staffing levels and performance of a group based on what information is required.

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