

## The Chaotic Economic Growth Model: China

Vesna D. Jablanovic<sup>1</sup>

### Abstract

Chaos theory reveals structure in aperiodic, dynamic systems. The number of nonlinear business cycle models use chaos theory to explain complex motion of the economy.

The basic aim of this paper is to provide a relatively simple chaotic economic growth model that is capable of generating stable equilibria, cycles, or chaos. This paper confirms stable economic growth in China in the period 1980-2012 (www.imf.org).

A key hypothesis of this work is based on the idea that the coefficient

$\pi = \left[ \frac{k + p_m(s - b - n)}{k} \right]$  plays a crucial role in explaining local stability of the gross

domestic product, where,  $s$  – private saving rate,  $p_m$  - marginal labour productivity coefficient,  $b$  – average budget deficit coefficient,  $k$  – capital coefficient and  $n$  – net capital outflow rate.

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**Keywords:** Economic growth, Economic growth of open economy, budget deficit

### 1 Introduction

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983), Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanovic (2010, 2011, 2012), etc.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the

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<sup>1</sup>Associate Professor of Economics, University of Belgrade, Serbia.

system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions; (ii) cause and effect are not proportional; and (iii) nonlinearity.

National saving is the source of the supply of loanable funds. Domestic investment and net capital outflow are the sources of the demand for loanable funds. At the equilibrium interest rate, the amount that people want to save exactly balances the amount that people want to borrow for the purpose of buying domestic capital and foreign capital. At the equilibrium interest rate, the amount that people want to save exactly balances the desired quantities of domestic investment and net capital outflow.

The supply and demand for loanable funds determine the real interest rate. The interest rate determine net capital outflow, which provides the supply of dollars in the market for foreign –currency exchange. The supply and demand for dollars in the market for foreign-currency exchange determine the real exchange rate.

In an open economy, government budget deficit raises real interest rates, crowds out domestic investment, decreases net capital outflow and causes the domestic currency to appreciate. However this appreciation makes domestic goods and services more expensive compared to foreign goods and services. In this case, exports fall, and imports rise. Namely, net exports fall. Further, the real gross domestic product falls as a consequence of the negative open economy multiplier effects

The basic aim of this analysis is to set up a relatively simple chaotic real gross domestic growth model that is capable of generating stable equilibria, cycles, or chaos. It is important to analyze the stability of the real gross domestic product growth in China in the period 1980-2012 ([www.imf.org](http://www.imf.org)).

## 2 A Simple Chaotic Economic Growth Model

The chaotic economic growth model is:

$$p_m = \frac{\Delta Y}{\Delta L} \quad (1)$$

$$S_t = S_{pt} - Bd_t \quad (2)$$

$$S_t = I_t + NCO_t \quad (3)$$

$$S_{pt} = s Y_t \quad (4)$$

$$I_t = K_{t+1} \quad (5)$$

$$Y_t = L_t^{1/2} \quad (6)$$

$$k = K_t / L_t \quad (7)$$

$$NCO_t = n Y_t \quad (8)$$

Where  $Y$  – the real gross domestic product ,  $I$  – investment,  $S$  – saving,  $S_p$  – private saving,  $Bd$  – budget deficit,  $L$  – labour,  $NCO$  – net capital outflow,  $K$  – capital,  $s$  – private saving rate,  $p_m$  – the marginal labour productivity coefficient,  $n$ - net capital outflow rate .

(1) defines the marginal labour productivity coefficient ; (2) defines national saving; (3) describes the identity between saving and investment plus net capital outflow; (4) determines the private saving; (5) determines investment; (6) contains the production function; (7) defines capital coefficient, and (8) determines net capital outflow.

By substitution one derives:

$$Y_{t+1} = \frac{k + p_m(s - b - n)}{k} Y_t - p_m Y_t^2 \tag{9}$$

Further, it is assumed that the current value of the gross domestic product is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the gross domestic product growth rate depends on the current size of the gross domestic product,  $Y$ , relative to its maximal size in its time series  $Y^m$ . We introduce  $y$  as  $y = Y/Y^m$ . Thus  $y$  range between 0 and 1. Again we index  $y$  by  $t$ , i.e., write  $y_t$  to refer to the size at time steps  $t = 0, 1, 2, 3, \dots$ . Now growth rate of the gross domestic product is measured as

$$y_{t+1} = \frac{k + p_m(s - b - n)}{k} y_t - p_m y_t^2 \tag{10}$$

This model given by equation (10) is called the logistic model. For most choices of  $k$ ,  $n$ ,  $s$ ,  $p_m$ , and  $b$  there is no explicit solution for (10). Namely, knowing  $k$ ,  $n$ ,  $s$ ,  $p_m$ ,  $b$ , and measuring  $y_0$  would not suffice to predict  $y_t$  for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz (1963) discovered this effect - the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (10) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of  $y_t$ . This difference equation (10) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point  $y_0$  the solution is highly sensitive to variations of the parameters  $k$ ,  $n$ ,  $s$ ,  $p_m$ , and  $b$ ; secondly, given the parameters  $k$ ,  $n$ ,  $s$ ,  $p_m$ , and  $b$ , the solution is highly sensitive to variations of the initial point  $y_0$ . In both cases the two solutions are for the first few periods rather close to each other, but later on they behave in a chaotic manner.

### 3 The Logistic Equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0, 4], \quad z_t \in [0, 1] \quad (11)$$

is equivalent to the iteration of growth model (10) when we use the identification

$$z_t = \left[ \frac{k p_m}{k + p_m (s - b - n)} \right] y_t \quad \text{and} \quad \pi = \left[ \frac{k + p_m (s - b - n)}{k} \right] \quad (12)$$

Using (10) and (12) we obtain:

$$\begin{aligned} z_{t+1} &= \left[ \frac{k p_m}{k + p_m (s - b - n)} \right] y_{t+1} = \\ &= \left[ \frac{k p_m}{k + p_m (s - b - n)} \right] \left\{ \left[ \frac{k + p_m (s - b - n)}{k} \right] y_t - p_m y_t^2 \right\} = \\ &= \left\{ p_m y_t - \left[ \frac{k p_m^2}{k + p_m (s - b - n)} \right] y_t^2 \right\} \end{aligned}$$

On the other hand, using (11) and (12) we obtain:

$$\begin{aligned} z_{t+1} &= \pi z_t (1 - z_t) = \\ &= \left[ \frac{k + p_m (s - b - n)}{k} \right] \left[ \frac{k p_m}{k + p_m (s - b - n)} \right] y_t \left\{ 1 - \left[ \frac{k p_m}{k + p_m (s - b - n)} \right] y_t \right\} \\ &= \left\{ p_m y_t - \left[ \frac{k p_m^2}{k + p_m (s - b - n)} \right] y_t^2 \right\} \end{aligned}$$

Thus we have that iterating  $y_{t+1} = \frac{k + p_m (s - b - n)}{k} y_t - p_m y_t^2$  is really the same as

iterating  $z_{t+1} = \pi z_t (1 - z_t)$  using  $z_t = \left[ \frac{k p_m}{k + p_m (s - b - n)} \right] y_t$  and

$\pi = \left[ \frac{k + p_m (s - b - n)}{k} \right]$ . It is important because the dynamic properties of the logistic equation (11) have been widely analyzed (Li and Yorke (1975), May (1976)).

It is obtained that :

- (i) For parameter values  $0 < \pi < 1$  all solutions will converge to  $z = 0$ ;
- (ii) For  $1 < \pi < 3,57$  there exist fixed points the number of which depends on  $\pi$ ;
- (iii) For  $1 < \pi < 2$  all solutions monotonically increase to  $z = (\pi - 1) / \pi$ ;
- (iv) For  $2 < \pi < 3$  fluctuations will converge to  $z = (\pi - 1) / \pi$ ;
- (v) For  $3 < \pi < 4$  all solutions will continuously fluctuate;
- (vi) For  $3,57 < \pi < 4$  the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means

that the path of  $z_t$  fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

### 4 Empirical Evidence

The main aim of this paper is to analyze the gross domestic product growth stability in the period 1980-2012, in China, by using the presented non-linear, logistic economic growth model (13):

$$y_{t+1} = \pi y_t - p_m y_t^2 \tag{13}$$

where  $y$  – the gross domestic product,  $\pi = \left[ \frac{k + p_m(s - b - n)}{k} \right]$ ,  $p_m$  – the coefficient of marginal productivity.

Firstly, data on the gross domestic product are transformed (www.imf.org) from 0 to 1, according to our supposition that actual value of the gross domestic product,  $Y$ , is restricted by its highest value in the time-series,  $Y^m$ . Further, we obtain time-series of  $y = Y / Y^m$ .

Table 1: The estimated model (13): China, 1980-2012. (Variance explained: 99.939%) (www.imf.org)

	$\pi$	$p_m$
<b>Estimate</b>	1.1062	.0064
<b>Std.Err.</b>	.0094	.01381
<b>t(29)</b>	118.0751	.463398
<b>p-level</b>	0.00000	0.646538

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### 5 Conclusion

This paper suggests conclusion for the use of the chaotic economic growth model in predicting the fluctuations of the gross domestic product. The model (10) has to rely on specified parameters  $k, n, s, p_m,$  and  $b,$  and initial value of the gross domestic product,  $y_0$ . But even slight deviations from the values of parameters:  $k, n, s, p_m,$  and  $b,$  and initial value of the gross domestic product,  $y_0$  show the difficulty of predicting a long-term economic behaviour.

A key hypothesis of this work is based on the idea that the coefficient  $\pi = \left[ \frac{k + p_m(s - b - n)}{k} \right]$  plays a crucial role in explaining local stability of the gross domestic product, where,  $s$  – private saving rate,  $p_m$  - marginal labour productivity coefficient,  $b$  – average budget deficit coefficient,  $k$  – capital coefficient and  $n$  – net capital outflow rate.

The estimated value of the coefficient  $\pi$  is greater than 1. This result confirms stable economic growth in China in the period 1980-2012. Increasing national saving and increasing marginal labour productivity are important ingredients in long-run economic growth in China.

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