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Portfolio Theory Forward Testing

Marcus Davidsson[1]

Abstract

Portfolio Theory has during many decades been considered as the holy grail of investment despite the fact that very few empirical studies in the public domain have shown that portfolio theory outperforms a random equal weighted portfolio. We will in this paper empirically investigate how successful portfolio theory is when it comes to generating large positive returns with low return volatility. The dataset that is used consists of approximately 4000 US stocks. We find weak support that portfolio theory by itself would have generated any returns different than a random portfolio allocation. In general optimized historical cumulative returns are not the same as forward cumulative returns.

JEL classification numbers: G00, G1,
Keywords: Portfolio theory, investment, finance

1 Introduction and Theory

Risk is something that can be quantified by using statistics. Uncertainty however is something that cannot be quantifiable (Knight, 1921). Uncertainty in information theory in the form of entropy has a little bit different meaning since it is directly related to risk (Shannon, 1951). Uncertainty, in the form of unpredictable outcomes, can also be found in deterministic (non stochastic) chaotic systems due to the so called Butterfly effect (Lorenz, 1963). Uncertainty in finance can be found both in the estimation of the expected return and in the estimation of the standard deviation of return i.e. both can change over time. It is also important to note that gambling and speculation are defined differently. Taleb (2007) explains that gambling takes place in a closed laboratory environment where the return distribution is known and where uncertainty is nonexistent. The expected return for a gambler is zero and remains constant over time. Risk and uncertainty in such a world is essentially parameterised. Speculation (Babusiaux *et al*, 2011) on the other hand takes place in an open environment where the future return distribution is not known and where

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[1]55316 Jönköping, SWEDEN
e-mail: davidsson_marcus@hotmail.com

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uncertainty is plentiful. The expected return for a speculator is undefined and it does not remain constant over time. The speculator is forced to use historical data to try to make inference about the shape of the return distribution in the future. Due to the large amount of uncertainty, the confidence interval that surrounds the speculator's decision making will become much larger than suggested by traditional statistics.

Portfolio theory was introduced to the world in six steps: Markowitz (1959), Sharpe (1964), Ross (1976), Black and Litterman (1992), Fama & French (1993) and Carhart (1997). The main objective for portfolio diversification is to minimize portfolio variance. Portfolio variance is a function of the return volatility for each security in the portfolio and the cross correlation of returns. Since cross correlation can be negative return variance can be cancelled out. However, the same idea can also be applied to highly positive correlated stock return portfolio by artificially creating negative cross correlation in return by short selling. Portfolio variance is the amount of return noise around the portfolio's expected return. Diversification can to a large extend eliminate such return noise. Markowitz (1959) mainly looks at diversification from an asset class perspective where an investor that spreads his risk between different asset classes will achieve a greater "diversification". Brinson *et al* (1986) found that asset class allocation (compared to market timing and stock picking) can explain on average 93.6 per cent of the variation in total return. It is also interesting to note that the bond returns in general tend to be the only return that will not become negative during a market crash (Longin and Solnik 1995). This means that bonds provides a good source of diversification due to return stability especially when markets has become more positive cross correlated during the last thirty years and even though the return on "risk free" government bonds has steadily been declining for the last 40 years.

The Capital Asset Pricing Model (CAPM) which was introduced by Sharpe (1964) points out that market risk also plays an important role for the smoothness of the equity curve. A portfolio with a large beta (i.e. highly sensitive to changes in market returns) will have more risk than a portfolio with a zero beta. An investor can reduce such market risk by balancing long and short positions. Market risk plays an important role when it comes to investing in financial markets because market returns accounts for a large fraction of stock returns (Fama and French, 1992). Ross (1976) introduced the so called Arbitrage Pricing Theory which illustrates that asset returns can be modelled as linear functions of various factor indices. Black and Litterman (1992) introduced the so called Black-Litterman model which starts by assuming that the benchmark index is mean-variance efficient and from such assumption derive the expected return of the benchmark portfolio. Fama & French (1993) introduced the three factor models which includes beta, book-to-market-ratio and stocks size which they claim will reduced return noise even further. Finally Carhart (1997) extend such a three-factor model to a four-factor model which also includes a momentum component which explains even more of the return variance. Conditional expected return also known as greed and conditional return volatility also known as fear are heavily used in portfolio theory i.e. the Sharpe ratio. Conrad & Kaul (1988) and Jegadeesh & Titman (1993) have found that conditional expected return is positive serial correlated and Mandelbrot (1963) and Engle (1982) have found that conditional return volatility is positive serial correlated. Serial correlation in returns tends to be insignificant (Runde & Kramer, 1991). Positive serial correlation in expected return and volatility is a contributing factor why we see price trends in financial markets. Even though we have positive serial correlation in the mean and volatility this is where most of the portfolio risk comes in. Portfolio rebalancing hence becomes the primary tool to minimize such risk (Karoglou, 2010) and (Powers, 2010). Previous studies such as Mandelbrot (1963) and Fama (1965) have also shown that finacial makrets tend to have fatter tails and a larger amount of kurtosis than the normal distribution.

Portfolio theory can also be understood by looking at the random walk model $S(t)=a+b*S(t-1)+R$ where R is an independent and identically distributed (i.i.d) random variable drawn from a normal distribution with mean μ and standard deviation σ, b takes a value of 1 and a is the drift coefficient i.e. expected return which can be either positive, negative or zero. The return for such a random walk model is given by $S(t)-S(t-1)=a+R(t)$ which means that the return for a random walk has two components; expected return a and random return noise $R(t)$. The random return noise $R(t)$ represents the fluctuations around the expected return a. The objective for most investors is to eliminate such random return noise $R(t)$ element through diversification. For a highly diversified portfolio the random return noise $R(t)$ is canceled out hence return becomes expected return a.

2 Empirics

In this section we will apply portfolio theory to empirical data. When we have a large global universe of stocks the easiest way to apply portfolio theory is to use least-squares. Such an approach is fast and can handle many 1000's of securities. We start by specifying the linear system. The linear system is given by **ER=R.W** where ER is a column vector containing the investor specified portfolio expected return for each time period which we assume is 2%, **R** is the return matrix and **W** is a weight vector. An error vector $\boldsymbol{r}$ is introduced hence the linear system can be written as **r=R.W - ER**. The objective becomes to minimize the sum of all the elements in **r**. Since we are only interested in the absolute error we minimize the sum of the square of which means that our objective function can be written as:

$$\min \|\mathbf{R.W} - \mathbf{ER}\|_2^2 .$$

The dataset consist of approximately 4000 US stocks. The dataset is split into three groups; S&P-SUPERCOMP (1115 stocks), NASDAQ (1415 stocks) and NYSE (1440 stocks). Each group is then split into back-testing and forward-testing data. Such a separation makes sure that we minimize curve fitting. We then apply statistical analysis on the back-tested and forward-tested sample. We test the hypothesis that the mean and standard deviation is the same for the two groups. In a perfect world the mean and standard deviation of the back-testing sample should be the same as the men and standard deviation of the forward-testing sample. However, as seen in table 1, 2, 3 and 4 there is quite a large difference. When you run the back-testing the equity curve is super smooth and upward sloping with an expected return equal to 2% and portfolio return variance close to zero. The forward-testing introduces a lot of volatility into the equity curve. In figure 1-4 we can see the expected return and portfolio variance of the forward-tested allocations. Table 5 to 7 contain the forward testing return correlation matrices. Table 8 contains the normality test and table 9 contains the simulated total return and the forward tested return.

Table 1: Statistical Analysis of S&P-SUPERCOMP (1115 stocks)

Standard T-Test on One Sample (Unknown Variance)					
Null Hypothesis: Sample drawn from population with mean 2					
Alt. Hypothesis: Sample drawn from population with mean not equal to 2					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX					
S&P-SUPERCOMP	Backtest Mean	Forward Test Mean	StudentT	P-Value	Outcome
BT 2000 FT=2001	2.000	0.207	-3.829	0.002	Rejected
BT 2000-2001 FT=2002	2.000	-2.745	-1.159	0.270	Accepted
BT 2000-2002 FT=2003	2.000	5.389	0.529	0.6	Accepted
BT 2000-2003 FT=2004	2.000	-0.594	-3.000	0.012	Rejected
BT 2000-2004 FT=2005	2.000	-1.558	-3.697	0.003	Rejected
BT 2000-2005 FT=2006	2.000	-2.591	-1.473	0.168	Accepted
BT 2000-2006 FT=2007	2.000	1.057	-0.823	0.427	Accepted
BT 2000-2007 FT=2008	2.000	1.447	-0.192	0.850	Accepted
BT 2000-2008 FT=2009	2.000	-0.893	-1.578	0.142	Accepted
BT 2000-2009 FT=2010	2.000	12.539	1.192	0.257	Accepted
BT 2000-2010 FT=2011	2.000	0.168	-0.532	0.605	Accepted
Chi-Square Test on One Sample					
Null Hypothesis: Sample drawn from population with standard deviation equal to 0.01					
Alt. Hypothesis: Sample drawn from population with standard deviation not equal to 0.01					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX					
S&P-SUPERCOMP	Backtest StDev	ForwardTest Stdev	ChiSquare	P-Value	Outcome
BT 2000 FT=2001	4.41*10^-15	1.620	289028.000	0.000	Rejected
BT 2000-2001 FT=2002	7.01*10^-15	14.182	2.21*10^7	0.000	Rejected
BT 2000-2002 FT=2003	7.80*10^-15	22.187	5.41*10^7	0.000	Rejected
BT 2000-2003 FT=2004	1.19*10^-14	2.995	986733	0.000	Rejected
BT 2000-2004 FT=2005	1.18*10^-14	3.333	1.22*10^6	0.000	Rejected
BT 2000-2005 FT=2006	7.99*10^-15	10.791	1.28*10^7	0.000	Rejected
BT 2000-2006 FT=2007	1.26*10^-14	3.966	1.73*10^6	0.000	Rejected
BT 2000-2007 FT=2008	1.49*10^-14	9.943	1.08*10^7	0.000	Rejected
BT 2000-2008 FT=2009	1.21*10^-14	6.350	4.43*10^6	0.000	Rejected
BT 2000-2009 FT=2010	1.64*10^-14	30.572	1.02*10^8	0.000	Rejected
BT 2000-2010 FT=2011	1.78*10^-14	11.922	1.56*10^7	0.000	Rejected

Table 2 : Statistical Analysis of NASDAQ (1415 stocks)

Standard T-Test on One Sample (Unknown Variance)					
Null Hypothesis: Sample drawn from population with mean 2					
Alt. Hypothesis: Sample drawn from population with mean not equal to 2					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXX					
NASDAQ	Backtest Mean	ForwardTest Mean	StudentT	P-Value	Outcome
BT 2000 FT=2001	2.000	-0.524	-5.161	0.000	Rejected
BT 2000-2001 FT=2002	2.000	0.046	-2.961	0.012	Rejected
BT 2000-2002 FT=2003	2.000	-2.532	-1.729	0.111	Accepted
BT 2000-2003 FT=2004	2.000	-214.417	-0.989	0.347	Accepted
BT 2000-2004 FT=2005	2.000	-0.561	-3.040	0.011	Rejected
BT 2000-2005 FT=2006	2.000	1.547	-0.474	0.644	Accepted
BT 2000-2006 FT=2007	2.000	-0.025	-1.354	0.202	Accepted
BT 2000-2007 FT=2008	2.000	1.269	-0.167	0.869	Accepted
BT 2000-2008 FT=2009	2.000	9.721	1.728	0.111	Accepted
BT 2000-2009 FT=2010	2.000	8.947	0.908	0.382	Accepted
BT 2000-2010 FT=2011	2.000	14.903	0.289	0.777	Accepted
Chi-Square Test on One Sample					
Null Hypothesis: Sample drawn from population with standard deviation equal to 0.01					
Alt. Hypothesis: Sample drawn from population with standard deviation not equal to 0.01					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXX					
NASDAQ	Backtest StDev	ForwardTest Stdev	ChiSquare	P-Value	Outcome
BT 2000 FT=2001	2.67*10^-15	1.694	315822.000	0.000	Rejected
BT 2000-2001 FT=2002	4.69*10^-13	2.285	574458.000	0.000	Rejected
BT 2000-2002 FT=2003	4.80*10^-13	9.077	9.064*10^6	0.000	Rejected
BT 2000-2003 FT=2004	4.27*10^-13	764.681	6.432*10^6	0.000	Rejected
BT 2000-2004 FT=2005	6.34*10^-14	2.918	936924	0.000	Rejected
BT 2000-2005 FT=2006	1.52*10^-14	3.308	1.204*10^6	0.000	Rejected
BT 2000-2006 FT=2007	3.78*10^-14	5.180	2.95*10^6	0.000	Rejected
BT 2000-2007 FT=2008	2.84*10^-14	15.077	2.50*10^7	0.000	Rejected
BT 2000-2008 FT=2009	1.43*10^-13	15.477	2.63*10^7	0.000	Rejected
BT 2000-2009 FT=2010	1.10*10^-13	26.485	7.71*10^7	0.000	Rejected
BT 2000-2010 FT=2011	1.08*10^-13	154.187	2.61*10^9	0.000	Rejected

Table 3 : Statistical Analysis of NYSE (1440 stocks)

Standard T-Test on One Sample (Unknown Variance)					
Null Hypothesis: Sample drawn from population with mean 2					
Alt. Hypothesis: Sample drawn from population with mean not equal to 2					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXX					
NYSE	Backtest Mean	ForwardTest Mean	StudentT	P-Value	Outcome
BT 2000 FT=2001	2.000	0.532	-2.251	0.045	Rejected
BT 2000-2001 FT=2002	2.000	-0.796	-5.262	0.000	Rejected
BT 2000-2002 FT=2003	2.000	-3.041	-3.747	0.000	Rejected
BT 2000-2003 FT=2004	2.000	-1.334	-3.825	0.002	Rejected
BT 2000-2004 FT=2005	2.000	-2.148	-2.469	0.03	Rejected
BT 2000-2005 FT=2006	2.000	1.214	-0.337	0.742	Accepted
BT 2000-2006 FT=2007	2.000	-2.975	-0.808	0.435	Accepted
BT 2000-2007 FT=2008	2.000	3.810	0.397	0.698	Accepted
BT 2000-2008 FT=2009	2.000	-5.972	-1.365	0.199	Accepted
BT 2000-2009 FT=2010	2.000	0.285	-1.077	0.304	Accepted
BT 2000-2010 FT=2011	2.000	-1.418	-0.882	0.396	Accepted

Chi-Square Test on One Sample					
Null Hypothesis: Sample drawn from population with standard deviation equal to 0.01					
Alt. Hypothesis: Sample drawn from population with standard deviation not equal to 0.01					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXX					
NYSE	Backtest StDev	ForwardTest Stdev	ChiSquare	P-Value	Outcome
BT 2000 FT=2001	2.65*10^-15	2.251	557574.000	0.000	Rejected
BT 2000-2001 FT=2002	5.29*10^-15	1.840	372815.000	0.000	Rejected
BT 2000-2002 FT=2003	6.51*10^-13	4.660	2.38*10^6	0.000	Rejected
BT 2000-2003 FT=2004	1.81*10^-13	3.019	1.00*10^6	0.000	Rejected
BT 2000-2004 FT=2005	5.00*10^-13	5.818	3.72*10^6	0.000	Rejected
BT 2000-2005 FT=2006	9.18*10^-15	8.064	7.15*10^6	0.000	Rejected
BT 2000-2006 FT=2007	1.52*10^-14	21.317	4.99*10^6	0.000	Rejected
BT 2000-2007 FT=2008	1.92*10^-13	15.766	2.73*10^7	0.000	Rejected
BT 2000-2008 FT=2009	2.58*10^-14	20.231	4.50*10^7	0.000	Rejected
BT 2000-2009 FT=2010	1.91*10^-13	5.512	3.34*10^6	0.000	Rejected
BT 2000-2010 FT=2011	6.63*10^-14	13.424	1.98*10^6	0.000	Rejected

Table 4 : Statistical Analysis of All Data (approx 4000 stocks)

Standard Z-Test on One Sample (Known Variance)					
Null Hypothesis: Sample drawn from population with mean 2 and known standard deviation 1 (the backtested standard deviation is close to zero so a standard deviation of 1 is generous)					
Alt. Hypothesis: Sample drawn from population with mean not equal to 2 and known standard deviation 1					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX					
NYSE	Backtest Mean	ForwardTest Mean	Statistics	P-Value	Outcome
BT 2000 FT=2001	2.000	0.207	-6.207	5.3*10^-10	Rejected
BT 2000-2001 FT=2002	2.000	-2.745	-16.440	9.8*10^-61	Rejected
BT 2000-2002 FT=2003	2.000	5.389	11.742	7.6*10^-32	Rejected
BT 2000-2003 FT=2004	2.000	-0.594	-8.986	2.5*10^-19	Rejected
BT 2000-2004 FT=2005	2.000	-1.558	-12.325	6.5*10^-35	Rejected
BT 2000-2005 FT=2006	2.000	-2.591	-15.905	5.8*10^-57	Rejected
BT 2000-2006 FT=2007	2.000	1.057	-3.264	0.0010	Rejected
BT 2000-2007 FT=2008	2.000	1.447	-1.913	0.055	Accepted
BT 2000-2008 FT=2009	2.000	-0.893	-10.021	1.2*10^-23	Rejected
BT 2000-2009 FT=2010	2.000	12.539	36.511	7.4*10^-89	Rejected
BT 2000-2010 FT=2011	2.000	0.168	-6.343	2.2*10^-10	Rejected
NASDAQ	Backtest Mean	ForwardTest Mean	Statistics	P-Value	Outcome
BT 2000 FT=2001	2.000	-0.524	-8.746	2.1*10^-18	Rejected
BT 2000-2001 FT=2002	2.000	0.046	-6.768	1.3*10^-11	Rejected
BT 2000-2002 FT=2003	2.000	-2.532	-15.701	1.4*10^-55	Rejected
BT 2000-2003 FT=2004	2.000	-214.417	-749.691	0	Rejected
BT 2000-2004 FT=2005	2.000	-0.561	-8.874	7.0*10^-19	Rejected
BT 2000-2005 FT=2006	2.000	1.547	-1.568	0.116	Accepted
BT 2000-2006 FT=2007	2.000	-0.025	-7.016	2.2*10^-12	Rejected
BT 2000-2007 FT=2008	2.000	1.269	-2.530	0.011	Rejected
BT 2000-2008 FT=2009	2.000	9.721	26.747	1.3*10^-9	Rejected
BT 2000-2009 FT=2010	2.000	8.947	24.068	5.4*10^-98	Rejected
BT 2000-2010 FT=2011	2.000	14.903	44.700	0	Rejected
NYSE	Backtest Mean	ForwardTest Mean	Statistics	P-Value	Outcome

BT 2000 FT=2001	2.000	0.532	-5.084	3.6*10^-7	Rejected
BT 2000-2001 FT=2002	2.000	-0.796	-9.688	3.3*10^-22	Rejected
BT 2000-2002 FT=2003	2.000	-3.041	-17.463	2.7*10^-68	Rejected
BT 2000-2003 FT=2004	2.000	-1.334	-11.552	7.1*10^-31	Rejected
BT 2000-2004 FT=2005	2.000	-2.148	-14.370	7.9*10^-47	Rejected
BT 2000-2005 FT=2006	2.000	1.214	-2.722	0.006	Rejected
BT 2000-2006 FT=2007	2.000	-2.975	-17.236	1.4*10^-66	Rejected
BT 2000-2007 FT=2008	2.000	3.810	6.271	2.5*10^-10	Rejected
BT 2000-2008 FT=2009	2.000	-5.972	-27.616	7.0*10^-94	Rejected
BT 2000-2009 FT=2010	2.000	0.285	-5.938	2.8*10^-9	Rejected
BT 2000-2010 FT=2011	2.000	-1.418	-11.841	2.3*10^-32	Rejected

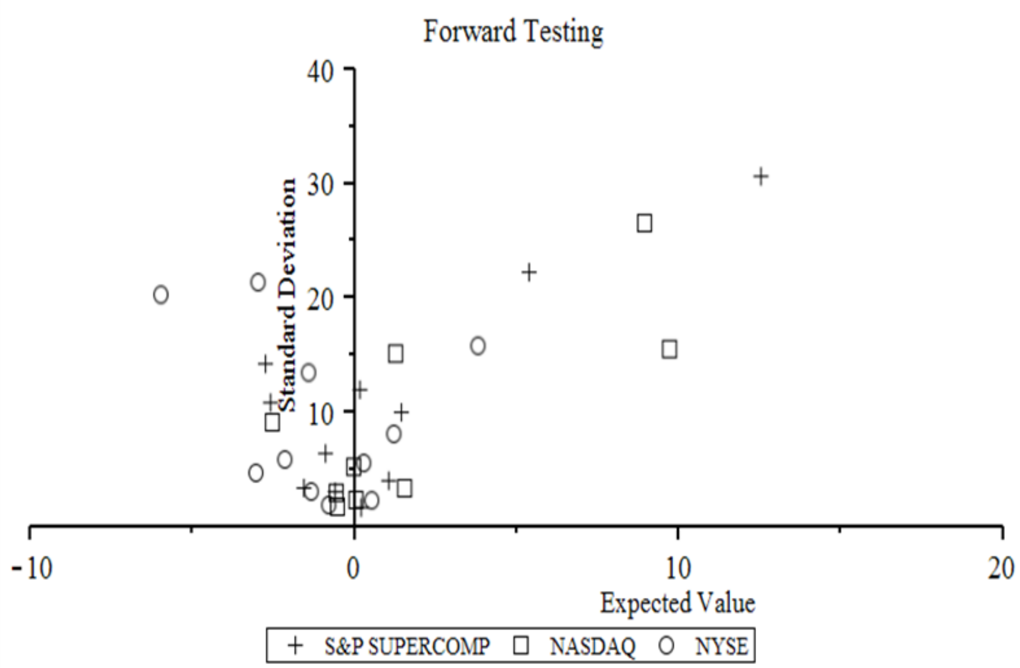

Figure 1: Expected Value and Standard Deviation Forward Testing

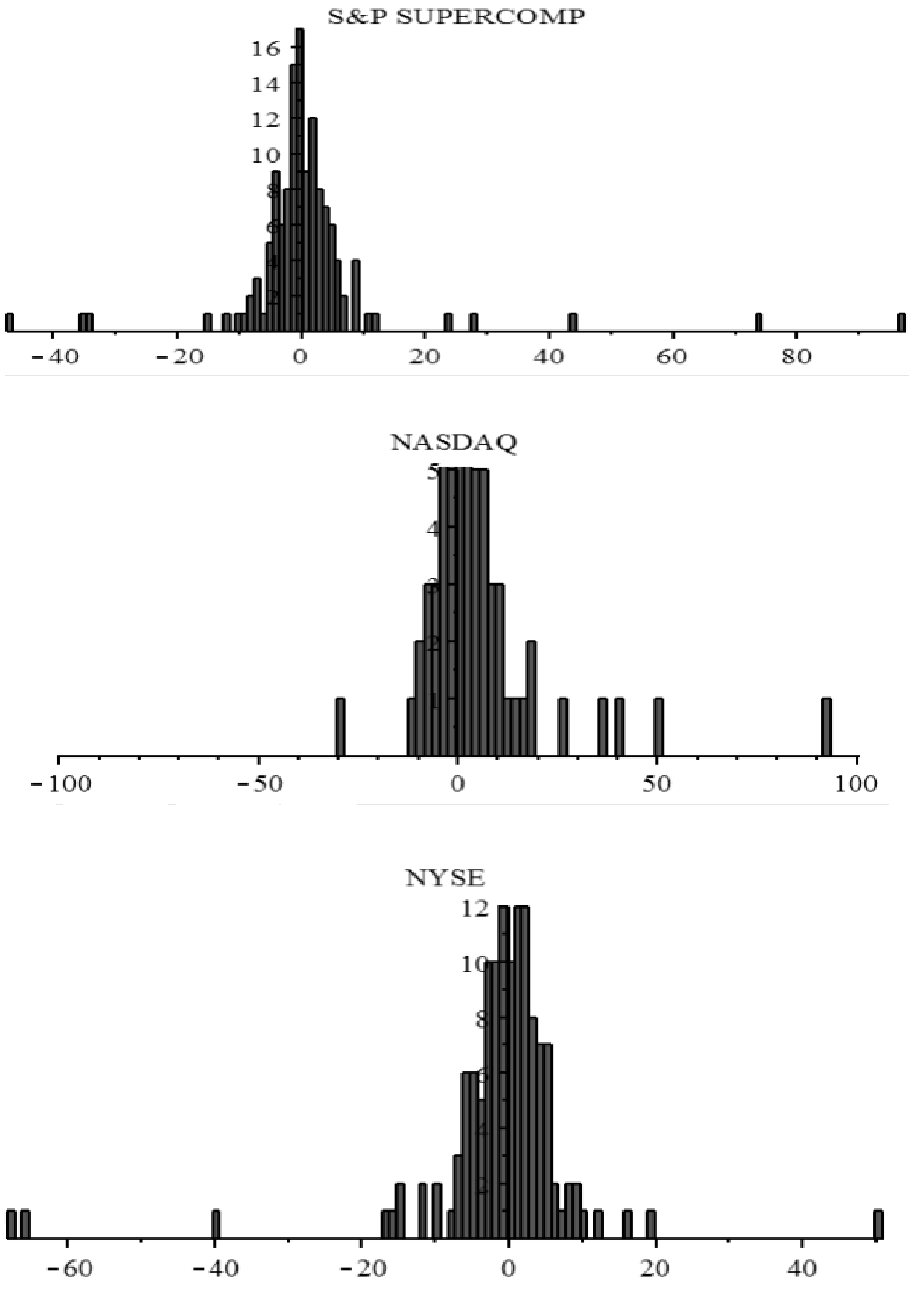

Figure 2: Forward Return Distributions

Table 5 : Correlation Matrix Forward Return S&P and NASDAQ

1.00	0.33	0.13	-0.15	-0.21	0.34	0.21	0.21	0.07	-0.07	0.38	-0.18	-0.54	0.13	0.37	-0.08	0.19	-0.06	-0.14	0.10	0.18	0.15
0.33	1.00	0.07	-0.08	-0.17	0.96	-0.18	0.33	0.61	0.05	-0.06	0.18	0.10	-0.19	-0.06	-0.16	-0.28	0.33	0.04	-0.13	0.14	0.08
0.13	0.07	1.00	-0.37	0.53	0.18	-0.01	0.15	-0.08	-0.15	0.29	0.05	0.41	0.23	0.04	-0.62	-0.47	0.59	0.59	-0.13	0.95	-0.01
-0.15	-0.08	-0.37	1.00	-0.15	-0.06	0.36	0.10	0.33	0.74	-0.09	0.20	-0.36	0.54	-0.16	0.63	0.18	-0.27	-0.11	0.28	-0.43	0.19
-0.21	-0.17	0.53	-0.15	1.00	-0.22	-0.34	-0.31	-0.34	-0.03	-0.34	-0.45	0.45	-0.15	-0.47	-0.32	0.14	-0.05	-0.01	-0.50	0.50	-0.12
0.34	0.96	0.18	-0.06	-0.22	1.00	-0.05	0.52	0.61	0.15	0.06	0.29	0.00	-0.14	0.06	-0.20	-0.41	0.49	0.27	-0.10	0.26	0.07
0.21	-0.18	-0.01	0.36	-0.34	-0.05	1.00	0.31	-0.12	0.00	0.14	0.38	-0.37	0.35	0.09	0.53	0.03	0.17	0.39	0.51	0.04	0.37
0.21	0.33	0.15	0.10	-0.31	0.52	0.31	1.00	0.43	0.39	0.23	0.21	-0.34	0.21	0.18	-0.15	-0.10	0.69	0.67	-0.21	0.16	0.02
0.07	0.61	-0.08	0.33	-0.34	0.61	-0.12	0.43	1.00	0.52	0.27	0.54	0.11	0.13	0.18	-0.05	-0.28	0.31	0.12	0.12	-0.05	0.25
-0.07	0.05	-0.15	0.74	-0.03	0.15	0.00	0.39	0.52	1.00	0.20	0.20	-0.38	0.33	0.20	0.21	-0.06	-0.06	0.01	-0.14	-0.15	-0.10
0.38	-0.06	0.29	-0.09	-0.34	0.06	0.14	0.23	0.27	0.20	1.00	0.52	-0.12	0.27	0.90	-0.35	-0.49	0.10	0.16	0.33	0.31	0.26
-0.18	0.18	0.05	0.20	-0.45	0.29	0.38	0.21	0.54	0.20	0.52	1.00	0.25	0.03	0.48	0.10	-0.70	0.32	0.35	0.54	0.16	0.40
-0.54	0.10	0.41	-0.36	0.45	0.00	-0.37	-0.34	0.11	-0.38	-0.12	0.25	1.00	-0.27	-0.29	-0.42	-0.37	0.25	0.16	-0.04	0.39	0.20
0.13	-0.19	0.23	0.54	-0.15	-0.14	0.35	0.21	0.13	0.33	0.27	0.03	-0.27	1.00	0.00	0.16	0.02	0.12	0.24	0.29	-0.01	0.07
0.37	-0.06	0.04	-0.16	-0.47	0.06	0.09	0.18	0.18	0.20	0.90	0.48	-0.29	0.00	1.00	-0.20	-0.47	-0.03	0.00	0.24	0.14	0.04
-0.08	-0.16	-0.62	0.63	-0.32	-0.20	0.53	-0.15	-0.05	0.21	-0.35	0.10	-0.42	0.16	-0.20	1.00	0.34	-0.37	-0.32	0.23	-0.59	-0.11
0.19	-0.28	-0.47	0.18	0.14	-0.41	0.03	-0.10	-0.28	-0.06	-0.49	-0.70	-0.37	0.02	-0.47	0.34	1.00	-0.46	-0.43	-0.23	-0.53	0.00
-0.06	0.33	0.59	-0.27	-0.05	0.49	0.17	0.69	0.31	-0.06	0.10	0.32	0.25	0.12	-0.03	-0.37	-0.46	1.00	0.90	-0.12	0.58	-0.05
-0.14	0.04	0.59	-0.11	-0.01	0.27	0.39	0.67	0.12	0.01	0.16	0.35	0.16	0.24	0.00	-0.32	-0.43	0.90	1.00	0.06	0.58	0.13
0.10	-0.13	-0.13	0.28	-0.50	-0.10	0.51	-0.21	0.12	-0.14	0.33	0.54	-0.04	0.29	0.24	0.23	-0.23	-0.12	0.06	1.00	-0.11	0.67
0.18	0.14	0.95	-0.43	0.50	0.26	0.04	0.16	-0.05	-0.15	0.31	0.16	0.39	-0.01	0.14	-0.59	-0.53	0.58	0.58	-0.11	1.00	0.04
0.15	0.08	-0.01	0.19	-0.12	0.07	0.37	0.02	0.25	-0.10	0.26	0.40	0.20	0.07	0.04	-0.11	0.00	-0.05	0.13	0.67	0.04	1.00

Table 6 : Correlation Matrix Forward Return NASDAQ and NYSE

1.00	0.25	0.03	0.48	0.10	-0.70	0.32	0.35	0.54	0.16	0.40	-0.12	-0.46	-0.20	0.10	-0.27	0.06	0.03	-0.15	-0.12	0.74	-0.26
0.25	1.00	-0.27	-0.29	-0.42	-0.37	0.25	0.16	-0.04	0.39	0.20	-0.36	-0.39	-0.45	0.22	-0.57	-0.51	-0.40	-0.15	-0.29	0.45	0.09
0.03	-0.27	1.00	0.00	0.16	0.02	0.12	0.24	0.29	-0.01	0.07	-0.26	0.22	-0.22	0.29	-0.01	0.23	0.02	0.17	-0.01	-0.11	-0.36
0.48	-0.29	0.00	1.00	-0.20	-0.47	-0.03	0.00	0.24	0.14	0.04	-0.10	-0.09	0.06	0.20	-0.32	-0.04	0.03	0.21	-0.13	0.18	-0.06
0.10	-0.42	0.16	-0.20	1.00	0.34	-0.37	-0.32	0.23	-0.59	-0.11	0.10	-0.30	0.31	-0.32	0.53	0.17	0.31	0.16	0.24	0.21	0.19
-0.70	-0.37	0.02	-0.47	0.34	1.00	-0.46	-0.43	-0.23	-0.53	0.00	0.43	0.20	0.35	-0.38	0.37	0.04	-0.01	0.20	0.17	-0.52	0.25
0.32	0.25	0.12	-0.03	-0.37	-0.46	1.00	0.90	-0.12	0.58	-0.05	0.25	0.36	-0.67	0.11	-0.25	0.54	0.12	-0.38	0.03	0.29	-0.76
0.35	0.16	0.24	0.00	-0.32	-0.43	0.90	1.00	0.06	0.58	0.13	0.20	0.45	-0.64	0.10	-0.12	0.57	0.17	-0.15	0.02	0.25	-0.90
0.54	-0.04	0.29	0.24	0.23	-0.23	-0.12	0.06	1.00	-0.11	0.67	-0.01	-0.21	0.23	0.16	0.33	0.17	0.35	0.31	0.32	0.13	0.02
0.16	0.39	-0.01	0.14	-0.59	-0.53	0.58	0.58	-0.11	1.00	0.04	-0.18	0.15	-0.77	0.72	-0.37	0.04	0.20	-0.08	0.21	-0.02	-0.27
0.40	0.20	0.07	0.04	-0.11	0.00	-0.05	0.13	0.67	0.04	1.00	0.24	-0.09	0.04	0.08	0.07	-0.04	-0.06	-0.08	0.09	-0.08	-0.03
-0.12	-0.36	-0.26	-0.10	0.10	0.43	0.25	0.20	-0.01	-0.18	0.24	1.00	0.50	0.09	-0.50	0.42	0.53	0.20	-0.24	0.26	-0.27	-0.24
-0.46	-0.39	0.22	-0.09	-0.30	0.20	0.36	0.45	-0.21	0.15	-0.09	0.50	1.00	-0.22	-0.22	0.26	0.42	0.01	-0.01	-0.03	-0.53	-0.54
-0.20	-0.45	-0.22	0.06	0.31	0.35	-0.67	-0.64	0.23	-0.77	0.04	0.09	-0.22	1.00	-0.40	0.52	0.04	0.13	0.10	0.14	-0.25	0.45
0.10	0.22	0.29	0.20	-0.32	-0.38	0.11	0.10	0.16	0.72	0.08	-0.50	-0.22	-0.40	1.00	-0.25	-0.09	0.33	0.10	0.42	-0.18	0.18
-0.27	-0.57	-0.01	-0.32	0.53	0.37	-0.25	-0.12	0.33	-0.37	0.07	0.42	0.26	0.52	-0.25	1.00	0.53	0.64	0.10	0.62	-0.44	0.11
0.06	-0.51	0.23	-0.04	0.17	0.04	0.54	0.57	0.17	0.04	-0.04	0.53	0.42	0.04	-0.09	0.53	1.00	0.68	-0.10	0.56	-0.12	-0.59
0.03	-0.40	0.02	0.03	0.31	-0.01	0.12	0.17	0.35	0.20	-0.06	0.20	0.01	0.13	0.33	0.64	0.68	1.00	0.25	0.93	-0.18	-0.01
-0.15	-0.15	0.17	0.21	0.16	0.20	-0.38	-0.15	0.31	-0.08	-0.08	-0.24	-0.01	0.10	0.10	0.10	-0.10	0.25	1.00	0.14	0.03	0.07
-0.12	-0.29	-0.01	-0.13	0.24	0.17	0.03	0.02	0.32	0.21	0.09	0.26	-0.03	0.14	0.42	0.62	0.56	0.93	0.14	1.00	-0.38	0.20
0.74	0.45	-0.11	0.18	0.21	-0.52	0.29	0.25	0.13	-0.02	-0.08	-0.27	-0.53	-0.25	-0.18	-0.44	-0.12	-0.18	0.03	-0.38	1.00	-0.25
-0.26	0.09	-0.36	-0.06	0.19	0.25	-0.76	-0.90	0.02	-0.27	-0.03	-0.24	-0.54	0.45	0.18	0.11	-0.59	-0.01	0.07	0.20	-0.25	1.00

Table 7: Correlation Matrix Forward Return S&P and NYSE

1.00	0.33	0.13	-0.15	-0.21	0.34	0.21	0.21	0.07	-0.07	0.38	0.43	0.32	0.03	0.34	0.22	0.24	0.35	-0.09	0.48	-0.65	0.20
0.33	1.00	0.07	-0.08	-0.17	0.96	-0.18	0.33	0.61	0.05	-0.06	0.27	-0.02	-0.05	0.06	0.01	0.12	-0.05	-0.91	0.07	-0.11	0.12
0.13	0.07	1.00	-0.37	0.53	0.18	-0.01	0.15	-0.08	-0.15	0.29	-0.28	0.26	-0.82	0.73	-0.43	0.00	0.07	-0.04	0.09	-0.07	-0.33
-0.15	-0.08	-0.37	1.00	-0.15	-0.06	0.36	0.10	0.33	0.74	-0.09	-0.20	-0.28	0.29	-0.07	0.29	0.18	0.08	-0.08	0.04	0.07	-0.07
-0.21	-0.17	0.53	-0.15	1.00	-0.22	-0.34	-0.31	-0.34	-0.03	-0.34	-0.38	-0.06	-0.39	0.43	-0.25	-0.38	-0.11	0.03	0.03	-0.24	0.18
0.34	0.96	0.18	-0.06	-0.22	1.00	-0.05	0.52	0.61	0.15	0.06	0.30	0.10	-0.14	0.08	0.02	0.30	0.06	-0.89	0.10	-0.06	-0.12
0.21	-0.18	-0.01	0.36	-0.34	-0.05	1.00	0.31	-0.12	0.00	0.14	0.38	0.22	-0.24	-0.07	0.40	0.44	0.40	0.20	0.28	0.12	-0.40
0.21	0.33	0.15	0.10	-0.31	0.52	0.31	1.00	0.43	0.39	0.23	0.52	0.51	-0.29	-0.22	-0.08	0.63	-0.01	-0.48	-0.11	0.02	-0.79
0.07	0.61	-0.08	0.33	-0.34	0.61	-0.12	0.43	1.00	0.52	0.27	0.04	-0.40	0.06	0.09	-0.29	0.20	-0.10	-0.64	-0.04	0.29	-0.11
-0.07	0.05	-0.15	0.74	-0.03	0.15	0.00	0.39	0.52	1.00	0.20	-0.28	-0.25	0.20	0.08	-0.08	0.20	0.00	-0.26	-0.10	0.08	-0.20
0.38	-0.06	0.29	-0.09	-0.34	0.06	0.14	0.23	0.27	0.20	1.00	-0.18	-0.05	-0.19	0.40	-0.49	-0.09	-0.09	0.17	-0.18	0.14	-0.18
0.43	0.27	-0.28	-0.20	-0.38	0.30	0.38	0.52	0.04	-0.28	-0.18	1.00	0.50	0.09	-0.50	0.42	0.53	0.20	-0.24	0.26	-0.27	-0.24
0.32	-0.02	0.26	-0.28	-0.06	0.10	0.22	0.51	-0.40	-0.25	-0.05	0.50	1.00	-0.22	-0.22	0.26	0.42	0.01	-0.01	-0.03	-0.53	-0.54
0.03	-0.05	-0.82	0.29	-0.39	-0.14	-0.24	-0.29	0.06	0.20	-0.19	0.09	-0.22	1.00	-0.40	0.52	0.04	0.13	0.10	0.14	-0.25	0.45
0.34	0.06	0.73	-0.07	0.43	0.08	-0.07	-0.22	0.09	0.08	0.40	-0.50	-0.22	-0.40	1.00	-0.25	-0.09	0.33	0.10	0.42	-0.18	0.18
0.22	0.01	-0.43	0.29	-0.25	0.02	0.40	-0.08	-0.29	-0.08	-0.49	0.42	0.26	0.52	-0.25	1.00	0.53	0.64	0.10	0.62	-0.44	0.11
0.24	0.12	0.00	0.18	-0.38	0.30	0.44	0.63	0.20	0.20	-0.09	0.53	0.42	0.04	-0.09	0.53	1.00	0.68	-0.10	0.56	-0.12	-0.59
0.35	-0.05	0.07	0.08	-0.11	0.06	0.40	-0.01	-0.10	0.00	-0.09	0.20	0.01	0.13	0.33	0.64	0.68	1.00	0.25	0.93	-0.18	-0.01
-0.09	-0.91	-0.04	-0.08	0.03	-0.89	0.20	-0.48	-0.64	-0.26	0.17	-0.24	-0.01	0.10	0.10	0.10	-0.10	0.25	1.00	0.14	0.03	0.07
0.48	0.07	0.09	0.04	0.03	0.10	0.28	-0.11	-0.04	-0.10	-0.18	0.26	-0.03	0.14	0.42	0.62	0.56	0.93	0.14	1.00	-0.38	0.20
-0.65	-0.11	-0.07	0.07	-0.24	-0.06	0.12	0.02	0.29	0.08	0.14	-0.27	-0.53	-0.25	-0.18	-0.44	-0.12	-0.18	0.03	-0.38	1.00	-0.25
0.20	0.12	-0.33	-0.07	0.18	-0.12	-0.40	-0.79	-0.11	-0.20	-0.18	-0.24	-0.54	0.45	0.18	0.11	-0.59	-0.01	0.07	0.20	-0.25	1.00

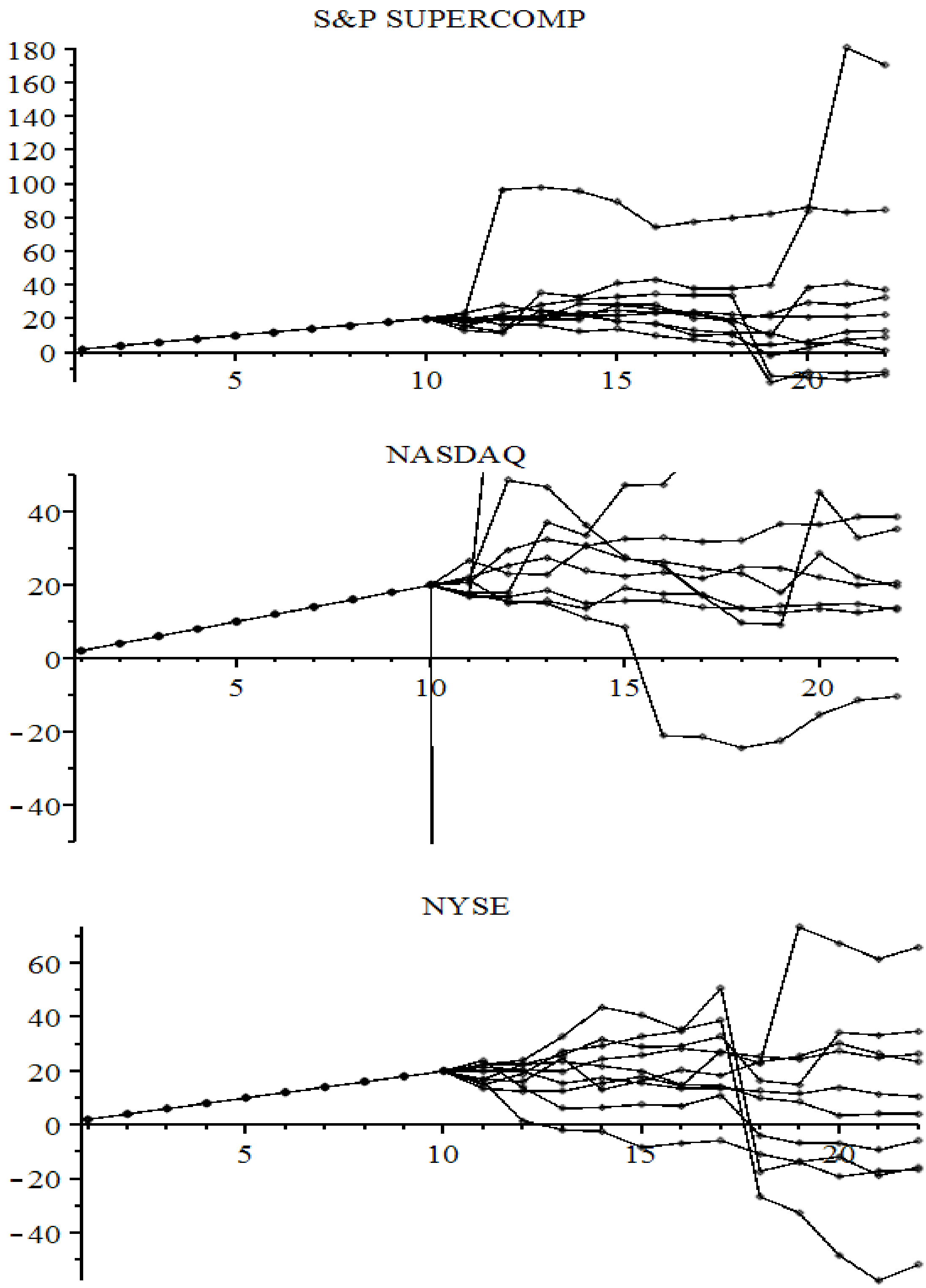

Figure 3: Back Testing and Forward Return Equity Curve

Table 8: Total Return, Risk Adjusted Return and Normality Test Forward Return

Shapiro and Wilk's W-Test for Normality					
Null Hypothesis: Sample drawn from a population that follows a normal distribution					
Alt. Hypothesis: Sample drawn from population that does not follow a normal distribution					
XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX XXXXXXXX					
NYSE	Total Return	Risk Adj.Return	Statistics	P-Value	Outcome
BT 2000 FT=2001	2.495	0.128	0.895	0.130	Accepted
BT 2000-2001 FT=2002	-32.951	-0.193	0.458	1.6*10^-6	Rejected
BT 2000-2002 FT=2003	64.678	0.242	0.536	9.3*10^-6	Rejected
BT 2000-2003 FT=2004	-7.130	-0.198	0.945	0.525	Accepted
BT 2000-2004 FT=2005	-18.697	-0.467	0.965	0.797	Accepted
BT 2000-2005 FT=2006	-31.096	-0.240	0.615	6.0*10^-5	Rejected
BT 2000-2006 FT=2007	12.691	0.266	0.933	0.389	Accepted
BT 2000-2007 FT=2008	17.371	0.145	0.819	0.013	Rejected
BT 2000-2008 FT=2009	-10.717	-0.140	0.976	0.926	Accepted
BT 2000-2009 FT=2010	150.477	0.410	0.693	0.000	Rejected
BT 2000-2010 FT=2011	2.026	0.014	0.717	0.000	Rejected
NASDAQ	Total Return	Risk Adj.Return	Statistics	P-Value	Outcome
BT 2000 FT=2001	-6.299	-0.309	0.950	0.598	Accepted
BT 2000-2001 FT=2002	0.552	0.020	0.925	0.315	Accepted
BT 2000-2002 FT=2003	-30.392	-0.279	0.665	0.000	Rejected
BT 2000-2003 FT=2004	-2573.006	-0.280	0.333	1.2*10^-7	Rejected
BT 2000-2004 FT=2005	-6.742	-0.192	0.943	0.508	Accepted
BT 2000-2005 FT=2006	18.565	0.467	0.919	0.269	Accepted
BT 2000-2006 FT=2007	-0.306	-0.004	0.869	0.061	Accepted
BT 2000-2007 FT=2008	15.233	0.084	0.763	0.002	Rejected
BT 2000-2008 FT=2009	116.654	0.628	0.801	0.008	Rejected
BT 2000-2009 FT=2010	107.374	0.337	0.467	2.0*10^-6	Rejected
BT 2000-2010 FT=2011	178.845	0.096	0.586	2.9*10^-5	Rejected
NYSE	Total Return	Risk Adj.Return	Statistics	P-Value	Outcome
BT 2000 FT=2001	6.387	0.236	0.937	0.435	Accepted
BT 2000-2001 FT=2002	-9.562	-0.432	0.957	0.697	Accepted
BT 2000-2002 FT=2003	-36.495	-0.652	0.863	0.050	Accepted
BT 2000-2003 FT=2004	-16.018	-0.442	0.941	0.476	Accepted
BT 2000-2004 FT=2005	-25.780	-0.369	0.887	0.103	Accepted
BT 2000-2005 FT=2006	14.570	0.150	0.852	0.036	Rejected
BT 2000-2006 FT=2007	-35.710	-0.139	0.572	2.1*10^-5	Rejected
BT 2000-2007 FT=2008	45.724	0.241	0.631	8.9*10^-5	Rejected
BT 2000-2008 FT=2009	-71.666	-0.295	0.686	0.000	Rejected
BT 2000-2009 FT=2010	3.428	0.051	0.942	0.494	Accepted
BT 2000-2010 FT=2011	17.021	-0.105	0.669	0.000	Rejected

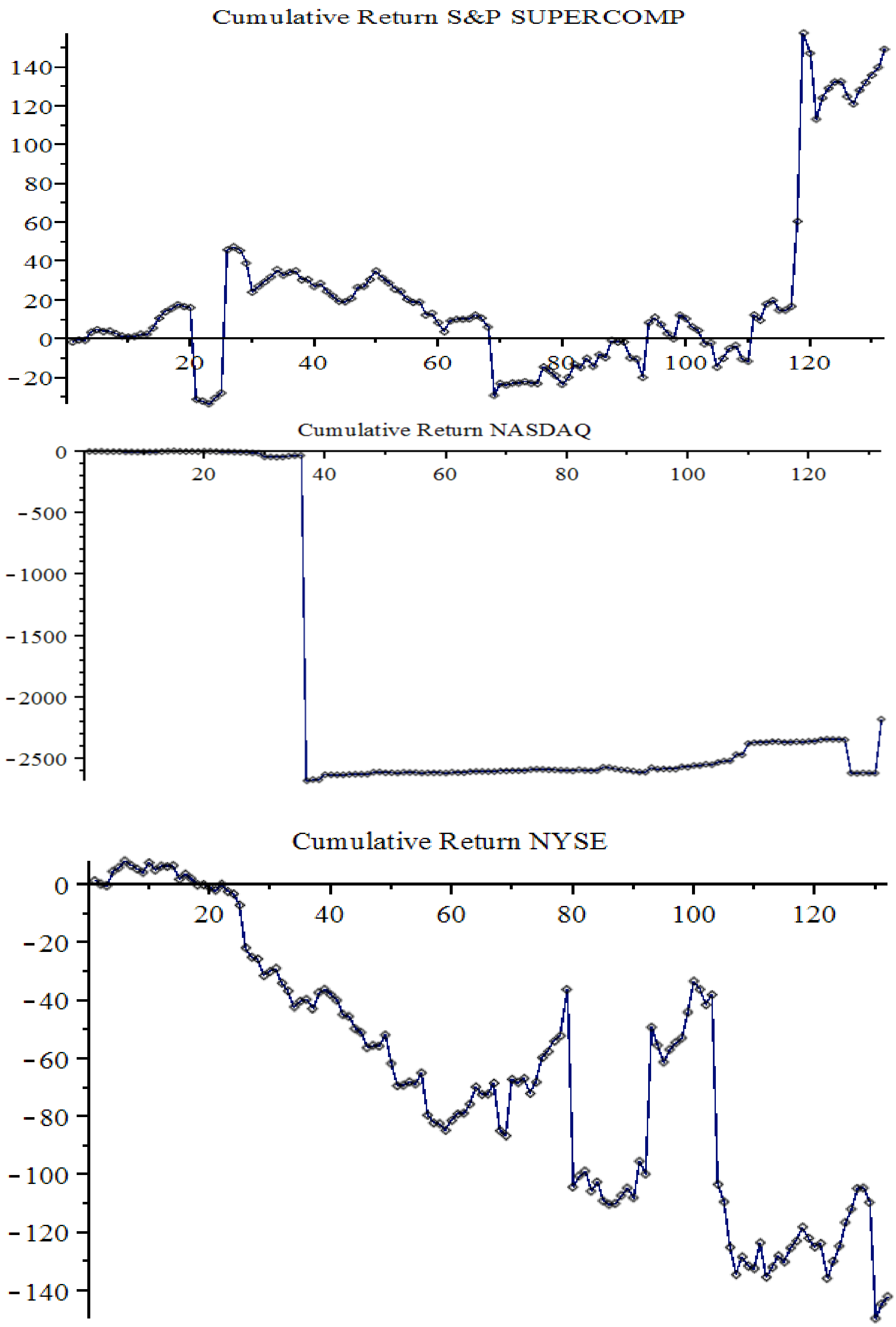

Figure 4: Cumulative Return Forward Testing

Table 9: Random Portfolio Returns vs. Optimized Portfolio Returns

NYSE	Expected Total Return 15 Random Allocations	80th Percentile	Total Return Optimized Portfolio	Outcome
BT 2000 FT=2001	156	417	2.495	Random
BT 2000-2001 FT=2002	-78	139	-32.951	Random
BT 2000-2002 FT=2003	24	321	64.678	Random
BT 2000-2003 FT=2004	-175	-4.3	-7.130	Random
BT 2000-2004 FT=2005	-13	251	-18.697	Random
BT 2000-2005 FT=2006	55.9	235	-31.096	Random
BT 2000-2006 FT=2007	-5.3	196	12.691	Random
BT 2000-2007 FT=2008	-6.5	199	17.371	Random
BT 2000-2008 FT=2009	-89	661	-10.717	Random
BT 2000-2009 FT=2010	31.3	257	150.477	Random
BT 2000-2010 FT=2011	28.9	142	2.026	Random
NASDAQ	Expected Total Return 15 Random Allocations	80th Percentile	Total Return Optimized Portfolio	Outcome
BT 2000 FT=2001	418	727	-6.299	Random
BT 2000-2001 FT=2002	-98	44	0.552	Random
BT 2000-2002 FT=2003	-418	167	-30.392	Random
BT 2000-2003 FT=2004	-437	196	-2573.006	Random
BT 2000-2004 FT=2005	53	470	-6.742	Random
BT 2000-2005 FT=2006	-159	409	18.565	Random
BT 2000-2006 FT=2007	-9	444	-0.306	Random
BT 2000-2007 FT=2008	-348	116	15.233	Random
BT 2000-2008 FT=2009	-117	1270	116.654	Random
BT 2000-2009 FT=2010	879	2120	107.374	Random
BT 2000-2010 FT=2011	-163	154	178.845	Not Random
NYSE	Expected Total Return 10 Random Allocations	80th Percentile	Total Return Optimized Portfolio	Outcome
BT 2000 FT=2001	-191	192	6.387	Random
BT 2000-2001 FT=2002	-18	190	-9.562	Random
BT 2000-2002 FT=2003	106	504	-36.495	Random
BT 2000-2003 FT=2004	13	199	-16.018	Random
BT 2000-2004 FT=2005	59	317	-25.780	Random
BT 2000-2005 FT=2006	8	188	14.570	Random

BT 2000-2006 FT=2007	30	226	-35.710	Random
BT 2000-2007 FT=2008	59	528	45.724	Random
BT 2000-2008 FT=2009	-169	100	-71.666	Random
BT 2000-2009 FT=2010	31	248	3.428	Random
BT 2000-2010 FT=2011	21	182	17.021	Random

3 Conclusion

We have in this paper used empirical data to try to answer the question; how successful is portfolio theory when it comes to generating large and stable returns? The hypothesis that the expected return was equal to 2% for the forward testing sample was accepted by the standard t-test. However the chi-square test indicated that the return volatility was far from zero. The more powerful z-test rejected the notion that the backward and forward sample where drawn from the same distribution. We have also found empirical support for the fact that portfolio theory's total returns was on par or worse than the total return generated by a random portfolio allocation. This can also be seen in the cumulative returns in figure-3. It now becomes interesting to discuss why such phenomenons were observed. The fact is that the majority of stocks do not have stable price trends that continue for decades at a time. This author speculate that a very large global universe i.e. >10 000 stocks might be required to find these very rare diamonds in the bush i.e. stable price trends. Portfolio theory is based upon very scientific principles and in theory portfolio theory works outstanding. However, in this case the empirical evidence was simply not there.

It is also worth pointing out that optimization *per se* is a somewhat romanticised science. Usually when someone uses the term optimized it implies that the outcome of such optimization will outperform i.e. if it would not outperform there would be no point in running the optimization. Such outperformance comes from the stable scientific foundation optimization rest on. However, sometimes stable scientific foundations are demolished by a simple fact that the expected return might be changing over time or even worse the expected return is not even positive to begin with. The historical cumulative return curve can be optimized to perfection i.e. an upward sloping straight line however when you take such an allocation and carry it into the future the same performance is not seen anymore. Two possible explanations; i) The future is truly uncertain which is something optimization never can capture. The optimization process is too perfect i.e. you need to introduce more randomness. ii) Our sample size was too small.

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Appendices

Appendix 1: Sample Portfolio Allocations

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0.00709	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0.000169	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.001724	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0.004222	0	0	0	0	0	0	0

Appendix 2: Efficient Frontier