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Time Series Models for Growth of Urban Population in SAARC Countries

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Abstract

The purpose of our present study is to strike out suitable models to explain the growth pattern and to forecast for urban population in SAARC countries. Using the data from UNPD for the years 1950 to 2000 in five years interval, we fitted both exponential and ARMA models. We found the superiority of ARMA models over exponential models to explain the time trend behavior of the urban population as a percentage of total population and to forecast up to the year 2025. We also found that urbanization is faster in Bangladesh than any other SAARC countries.

JEL classification numbers: C10, R11

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1 Introduction

Time trend behavior of population parameter has been extensively studied. To fit a time trend model at first graphical representation of the data is examined. The line graph of population parameters generally posses the exponential growth criterion (Misra, 1995; Bhende and Kanitkar 1997). Thus, to explain the growth pattern of population data exponential growth models were preferred by many authors (UN, 1967 and 1997; UNFPA, 1993; BBS, 2003). But, we can not assure that the data will be suitable for exponential modeling rather there might have some other modeling techniques that can be more suitable for. Time series data are time dependent and are autocorrelated (Pankratz, 1991; Gujarati, 1995; Cleary and Hay, 1980). So, we can apply the time series modeling techniques based on the effects of autocorrelation. In this paper, an attempt is made to examine the suitability of both exponential and time series models for the urban population in SAARC countries expressed as a percentage of total population.

2 Data and Methods

The data have been collected from UNPD (1999) and Earthtrend (2004). The data have been shown graphically using line plot (Figure1). We have analyzed the data using some sophisticated models including the time trend behavior of the data. As the time frame was in five years interval from 1950 to 2000 so there might have some impact over the passages of time. In this recognition we need to examine whether the data possess stationary criterion or not. We used the unit root test of MacKinnon (1996) to test the stationarity of the data.

An autoregressive model of order p, AR(p) (Cleary and Hay, 1980; Pankratz, 1991, Gujarati, 1995; and Hamilton, 1994) is of the form

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i Y_{t-i} + \gamma t + u_t \quad (1)$$

where, Y_t is the value of the variable Y (urban population as a percentage of total population) at time t, α is the intercept term, $\beta_i \, (i = 1, 2, ..., p)$ are the parameters, Y_{t-i} is the i-th lagged variable, t is the time trend, γ is the coefficient of the time variable t, and u_t is the error term which is a white noise indeed.

If we include the effect of q moving average terms, intercept, and trend component along with the AR(p) model then an autoregressive moving average model of order p and q, that is, ARMA(p,q/C,T) model (Gujarati, 1995 and Pankratz, 1991) can be formed as

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i Y_{t-i} + \sum_{j=1}^{q} \delta_j u_{t-j} + \gamma t + u_t \quad (2)$$

where α is the intercept term and t is the time trend. ARMA(p,q/C, T) refers to autoregressive moving average model with p autoregressive terms and q moving average terms including intercept term (C) and trend component (T).

For testing the significance of the fitted model stationarity criterion of residuals and outlier detection techniques (Pankratz, 1991) are applied. The stationarity of residuals are examined using the unit root test (MacKinnon, 1996). We have used the method of examining standardized residuals (absolute value of standardized residuals over 3.0 implies the presence of an outlier) to examine the presence of outlier (Pankratz, 1991). We have also used the recent developed cross validity predictive power (CVPP) and restricted cross validity predictive power

(RCVPP) (Khan and Ali, 2003a). The cross validity predictive power (CVPP) due to Stevens (1996) is

$$\rho^2_{cv} = 1 - w(1 - R^2); \quad w = \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)} \tag{3}$$

and the restricted cross validity predictive power is

$$\rho^2_{rcv} = \begin{cases} 1 - w(1 - R^2); \; R^2 \geq 1 - w^{-1}, \; w = \dfrac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}, \; n > k+2 \\ 0; \text{ otherwise.} \end{cases} \tag{4}$$

where R^2 is the coefficient of multiple determination, n is the sample size, k is the number of regressors used in the model, and $\frac{n+1}{n} \leq w \leq \frac{(n+1)(n-1)(n-2)}{2n}$.

Further, the shrinkage of R^2 in computing RCVPP has been computed from the absolute difference between RCVPP and R^2 (Stevens, 1996; and Khan and Ali, 2003a), that is,

$$\eta = \left|\rho^2_{rcv} - R^2\right| = \left|1 - w(1 - R^2) - R^2\right| \tag{5}$$

Now, $\rho^2_{rcv} = 0.95$ indicates that if we fit the same model to some other data from the same population then the fitted model will be able to explain 95% variation of the dependent variable. Further, $\eta = 0.01$ indicates that over the population the fitted model is 99% stable.

Thereafter, we have forecasted from the fitted models and have computed the root mean squared forecast error to examine the forecasting performance of the fitted model. Root mean squared forecast error is computed (Pankratz, 1991) using the formula

$$\text{RMSFE} = \sqrt{\frac{\sum_{t=n+1}^{n+r} (O_t - F_t)^2}{r}} \tag{6}$$

where O_t is the observed value, F_t is the corresponding forecasted value, and r is the number of periods to forecast.

Unfortunately, these observed values, O_t 's are not always in existence. So, we have used the estimated root mean squared forecast error (ERMSFE) to examine the forecasting performance of the fitted model (Khan and Ali, 2003b) that is computed as

$$ERMSFE = \sqrt{\frac{(n+r)(n+r-k-1)(n+r-k-2)}{r(n+r-1)(n+r-2)(n+r+1)} SS_{n+r}(t)(1-\rho^2_{rcv}) - \frac{SS(e)}{r}} \quad (7)$$

where, $SS_{n+r}(t)$ is the sum of squared total of n observed values and r forecasted values, $SS_n(t) = SS(total)$ is the sum of squared total of n observed values, $SS(e)$ is the sum of squared residuals, and k is the number of predictors used in the model.

3 Results and Discussions

In 1950 urban population of Bangladesh was 4.2% of total population. But, in 2000 this percentage becomes 24.5, that is, within 50 years the increment of urban population is 20.3%. Within these 50 years (1950-2000) urban population in India, Pakistan, Sri Lanka, Nepal, Bhutan, and Maldives lifted up 11.1%, 19.5%, 9.2%, 9.6%, 5.0%, and 15.5%, respectively. If we order the country with respect to the increment of urban population within 1950 to 2000 then we can write BD>PK>ML>IN>NP>SL>BH. Thus, we can say that the urbanization is faster in Bangladesh than any other SAARC countries.

To fit time trend models we have examined the stationarity of the variables using the unit root test and have found that all the variables are stationary at level, that is, integrated of order zero. The estimated results are given at Table 1.

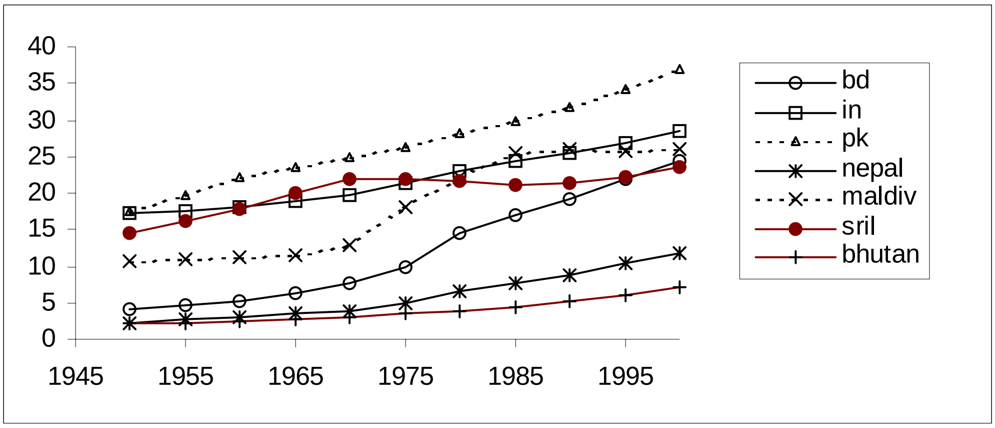

Figure1: Urban population (in percent of total population) in SAARC countries

Table 1: Unit Root Test

Variables	Specification	DF-Value	MacKinnon Critical	Stationary at
Bangladesh	None	5.764310	-2.8622*	Level
India	None	9.237166	-2.8622*	Level
Pakistan	None	12.31482	-2.8622*	Level
Nepal	None	11.02205	-2.8622*	Level
Bhutan	None	14.23454	-2.8622*	Level
Maldives	C, T and lag 1	-4.242453	-4.0815**	Level
Sri Lanka	None	2.420738	-1.9791**	Level

Here * (**) indicates values at 1% (5%) level.

We have fitted ARMA models to explain the time trend behavior of those variables (Table 2). Similarly, the fitted exponential growth models have shown in Table 3. From Table 2 and Table 3 we can say that the computed shrinkage of the fitted exponential growth model for urban population in Bangladesh, Nepal, Bhutan, and Maldives are less than their corresponding fitted ARMA models.

Similarly, the computed shrinkage of the fitted ARMA model for urban population as a percentage of total population in India, Pakistan, and Srilanka are less than their corresponding fitted exponential growth models.

Table 2: Fitted ARMA models for SAARC countries

Country & model	Fitted Models	R^2	RCVPP	Shrinkage
Bangladesh ARMA (1,0/C)	$\hat{X}_t = 2.111777T + 0.709472X_{t-1}$ $\Pr ob. = (0.00000)\ (0.0178)$ $Inverted\ AR = 0.71$	0.97606	0.96614	0.009916
India ARMA (1,0/C,T)	$\hat{X}_t = 12.16857 + 1.473118T + 0.629199X_{t-1}$ $\Pr ob. = (0.00001)\ (0.00000)\ (0.0014)$ $Inverted\ AR = 0.63$	0.99715	0.99464	0.002516
Pakistan ARMA (1,0/T)	$\hat{X}_t = 2.650389T + 0.937693X_{t-1}$ $\Pr ob. = (0.00000)\ (0.0000)$ $Inverted\ AR = 0.94$	0.99372	0.99112	0.002599
Sri Lanka ARMA (0,1/C,T)	$\hat{X}_t = 15.83038 + 0.771334T + 0.989817\hat{u}_{t-1}$ $\Pr ob. = (0.0000)\ (0.0002)\ (0.0000)$ $Inverted\ MA = -.99$	0.92595	0.86037	0.065584
Nepal ARMA (0,1/T)	$\hat{X}_t = 1.005028T + 0.920128\hat{u}_{t-1}$ $\Pr ob. = (0.00000)\ (0.0000)$ $Inverted\ MA = -.92$	0.98516	0.97203	0.013137
Bhutan ARMA (1,1/T)	$\hat{X}_t = 0.625149T + 0.659135X_{t-1} + 0.80589$ $\Pr ob. = (0.00000)\ (0.0087)\ \ (0.0107$ $Inverted\ AR = 0.66,\ \text{Inverted MA} = -0.$	0.98768	0.96748	0.020199
Maldives ARMA (1,0/T)	$\hat{X}_t = 2.493025T + 0.727347X_{t-1}$ $\Pr ob. = (0.0001)\ (0.0061)$ $Inverted\ AR = 0.73$	0.93481	0.87707	0.057739

Here RCVPP is restricted cross validity predictive power.

It is to note that the residual analysis (normality and outlier detection) were performed. No outlier was present in the data set as all the standardized residuals were within ± 3. We found that the residuals were normal with respect to their normal probability plot. For simplicity of content we would like to relax those graphical representations.

Table 3: Fitted Exponential growth models for SAARC countries

Country & model	Fitted Models	R^2	RCVPP	Shrinkage
Bangladesh	$\hat{X}_t = e^{1.261346 + 0.181728T}$ Prob. = (0.0000) (0.0000)	0.973273	0.963554	0.009719
India	$\hat{X}_t = e^{2.745215 + 0.054399T}$ Prob. = (0.0000) (0.0000)	0.987994	0.983628	0.004366
Pakistan	$\hat{X}_t = e^{2.857428 + 0.068223T}$ sig. = (0.0000) (0.0000)	0.992787	0.990164	0.002623
Sri Lanka	$\hat{X}_t = e^{2.785427 + 0.035485T}$ Prob. = (0.0000) (0.0011)	0.726173	0.626599	0.099573
Nepal	$\hat{X}_t = e^{0.632347 + 0.169864T}$ Prob. = (0.0000) (0.0000)	0.992601	0.989910	0.002690
Bhutan	$\hat{X}_t = e^{0.498468 + 0.129555T}$ Prob. = (0.0000) (0.0000)	0.990309	0.986785	0.003524
Maldives	$\hat{X}_t = e^{2.218693 + 0.105518T}$ Prob. = (0.0000) (0.0000)	0.879701	0.835956	0.043745

Here RCVPP is restricted cross validity predictive power.

Forecasted urban population in SAARC countries have shown in Table 4 and in Table 5. Table 5 shows the forecasted urban population as a percentage of total population from the fitted exponential models (Table 3) and Table 4 shows the forecasted values from the fitted ARMA models (Table 2). Also, the ERMSFE have been incorporated in those tables. From Table 4 and Table 5 we see that the ERMSFEs from the fitted ARMA models are comparatively smaller than those from the fitted exponential growth models. Fitted exponential model for the urban population of Nepal and Bhutan have greater RCVPP as well as greater ERMSFE than the corresponding fitted ARMA models. But, RMSFE decreases with the increase of restricted cross validity predictive power (as explained by Khan and Ali, 2003b).

Table 4: Forecasted urban population using the fitted models of Table 2

Year	Bangladesh	India	Pakistan	Nepal	Maldives	Sri Lanka	Bhutan
2005	26.24267 (2.25387)	29.86306 (0.75763)	39.16154 (1.42465)	12.51775 (0)	28.95382 (7.24813)	25.09739 (2.58357)	7.964004 (0.24594)
2010	28.09258 (2.31271)	31.32985 (0.65292)	41.35354 (1.24932)	13.06537 (0.33268)	31.70927 (5.89522)	25.85773 (2.14494)	8.431600 (0.31404)
2015	30.01857 (2.39157)	32.79898 (0.62362)	43.57411 (1.20350)	14.07040 (0.42105)	34.39317 (5.49477)	26.62906 (1.99391)	8.952901 (0.33781)
2020	31.99854 (2.47794)	34.26959 (0.61559)	45.82145 (1.19410)	15.07543 (0.46628)	37.02502 (5.37585)	27.4004 (1.92708)	9.5096 (0.35263)
2025	34.0168 (2.56901)	35.74113 (0.6163)	48.09391 (1.1999)	16.08045 (0.49694)	39.61902 (5.3740)	28.17173 (1.89667)	10.08963 (0.3643381)

Table 5: Forecasted urban population using the fitted models of Table 3

Year	Bangladesh	India	Pakistan	Nepal	Maldives	Sri Lanka	Bhutan
2005	31.2518 (3.433106)	29.9043 (1.01147)	39.4922 (1.23291)	14.45025 (0.80357)	32.61903 (5.72090)	24.8101 (2.96774)	7.792237 (0.42748)
2010	37.4799 (3.744152)	31.5762 (1.02514)	42.2805 (1.25882)	17.12561 (0.87193)	36.249 (5.92287)	25.7063 (2.91585)	8.870073 (0.45337)
2015	44.94917 (4.218421)	33.3415 (1.07606)	45.2656 (1.33041)	20.29631 (0.97633)	40.28291 (6.36453)	26.6349 (3.00487)	10.097 (0.49602)
2020	53.90698 (4.810053)	35.2054 (1.13988)	48.4615 (1.41913)	24.05403 (1.10554)	44.76574 (6.91482)	27.597 (3.13703)	11.49363 (0.54809)
2025	64.64997 (5.523798)	37.1736 (1.21163)	51.8831 (1.51933)	28.50747 (1.259875)	49.74744 (7.550968)	28.5938 (3.291503)	13.08345 (0.608558)

Here, values in parenthesis are estimated root mean squared forecast error (ERMSFE).

On the other hand, the fitted exponential models for urban population of Bangladesh, Nepal, Bhutan, and Maldives posses less shrinkages but more ERMSFEs compared to their associated fitted ARMA models. Table 4 and Table 5 depict that the ERMSFEs depend on both the initial population and the regression coefficient. If the initial population is greater but the regression coefficient is smaller then the ERMSFE becomes smaller and the ERMSFE becomes larger if the regression coefficient is larger. From Table 3 we find that the regression coefficient of the fitted exponential model for Bangladesh is greater than that for Pakistan and India and so the ERMSFE for Bangladesh is larger.

5 Conclusions

In fitting growth models for urban population in SAARC countries exponential models for Bangladesh, Nepal, Bhutan, and Maldives lead less shrinkage but more estimated root mean squared forecast error than the corresponding fitted ARMA models. But, ARMA models for all the seven SAARC countries lead acceptable shrinkage, smaller and more significantly accepted root mean squared forecast error than the fitted exponential growth models.

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References

[1] Bangladesh Bureau of Statistics, Statistical Year Book, Various Issues.

[2] A.A. Bhende and T. Kanitkar, *Principles of Population Studies*, Himalay Publishing House, Mumbai, 1997.

[3] Earthtrend (2004) http://earthtrends.wri.or

[4] D.N. Gujarati, *Basic Econometrics*, Mc Graw-Hill International, 3rd Edi. 1995.

[5] J.D. Hamilton, *Time Series Analysis*, Princeton University Press, *Princeton*, New Jersey, 1994.

[6] M.A.R. Khan and M.A. Ali, Restricted Cross Validity Predictive Power, *Pakistan Journal of Statistics*, **19**(2), (2003a).

[7] M.A.R. Khan and M.A. Ali, VAR Modeling with Mixed Series, *International Journal of Statistical Sciences*, **2**, (2003b).

[8] R.G. King, C.I. Plosser, J.H. Stock and M.W. Watson, Stochastic Trends and Economic Fluctuations, *Amer. Econ. Rev.*, 81, (September, 1991), 819-840.

[9] J.G. MacKinnon, Numerical Distribution Function for Unit Root and Cointegration Tests, *Journal of Applied Econometrics*, 11, (1996), 601-618.

[10] R. Mc Cleary and R.A. Hay, *Applied Time Series Analysis for the Social Science*, Sega Publications, California, 1980.

[11] B.D. Misra, *An Introduction to the Study of Population*, Second Edition, South Asian Publishers Pvt. Ltd, New Delhi, 1995.

[12] A. Pankratz, *Forecasting with Dynamic Regression Models*, John Wiley & Sons Inc., New York, 1991.

[13] J. Stevens, *Applied multivariate statistics for the social sciences*, Third Edition, Lawrence Erlbaum Associates, Inc., Publishers, Mahwah, New Jersey, 1996.

[14] UN., *The Determinant and Consequences of Population Trends*, Series A, Population Studies, **1**(42), New York, 1967.

[15] UN., *Fertility Trends Among Low Fertility Countries, Expert Group Meeting on Below-Replacement Fertility*, Population Division, *UN Secretariat*, New York, 1997.

[16] UNPD, United Nations Population Division, New York, 1999.

[17] UNFP, Readings in Population Research Methodology, **3** and **4**, (1993).