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Nonlinear Integer Programming Transportation Models: An Alternative Solution

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Abstract

The combinatorial nature of integer programming is inevitable even after taking specific model structure into consideration. This is the root problem in implementing large-scale nonlinear integer programming models regardless of which algorithm one chooses to use. Consequently, we suggest that the size of origin-destination be moderate. In the case of large origin-destination problems, more information on the size of x_{ij} is needed to drastically reduce the dimensionality problem. For instance, if x_{ij} is to be greater than the threshold value to be eligible for the rate break, computation time can be noticeably reduced. In the case of large right-hand-side constraints, we suggest scaling these

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values to the nearest thousands or millions. The approach from Excel proposed in this paper is particularly appropriate if one can balance the sizes of origin-destination and right-hand-side constraints in such a way that computation time is not excessive. For a large-scale problem, one must exploit the structure of the model and acquire more information on the bounds of discrete variables. Our approach certainly provides an alternative way to solve nonlinear integer programming models with virtually all kinds of algebraic functions even for laymen who do not feel comfortable with mathematic programming jargons.

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1 Introduction

Spatial interaction models have received a great deal of attention either in theoretical advances or empirical applications. In the early 1940's, spatial allocation problems were cast in the form of the linear programming transportation models (LPT) developed by Hitchcock [24], Kantorovich [30] and Koopmans [33]. The original works by Hitchcock [24] was published in mathematical physics as was that by Kantorovich [30]. The Koopmans' work was indeed the first on transportation modeling in economics [33]. And more recently Arsham and Khan [1] offered an alternative algorithm to the stepping stone method. Enke [11] laid the foundation of the spatial equilibrium model based on the Kirchhoff law of electrical circuits. Samuelson's influential work [59] on spatial price equilibrium (SPE) has generated a significant amount of interest in the spatial economics. In 1964, Takayama and Judge [64] reformulated the Enke-Samuelson problem into a quadratic programming model with the objective of maximizing "net social payoff." Since then, theoretical advances and refinements along the line of the Enke, Samuelson, Takayama and Judge abound.

There has been a large body of literature that improves on or extends the original Takayama-Judge model, including: reformulation and a new algorithm by Liew and Shim [44]; inclusion of income by Thore [66]; transshipment and location selection problem by Tobin and Friesz [67]; sensitivity analyses by Yang and Labys [76], Dafermos and Nagurney [7]; computational comparison by Meister, Chen and Heady [47]; iterative methods by Irwin and Yang [27]; a linear complementarity formulation by Takayama and Uri [65]; sensitivity analysis of complementarity problems by Yang and Labys [77]; applications of the linear complementarity model by Kennedy [32]; a solution condition by Smith [61]; the spatial equilibrium problem with flow dependent demand and supply by Smith and Friesz [62]; nonlinear complementarity models by Irwin and Yang [28] and Rutherford [57]; variational inequalities by Harker [19]; a path dependent spatial equilibrium model by Harker [20]; and dispersed spatial equilibrium by Harker [21]. In addition, the SPE model has become increasingly fused with other types of spatial models. For instance, the solutions of a SPE model can be obtained and combined with the gravity model (Harker [21]) and the commodity or passenger flows can also be estimated using econometric (e.g., the logit model, Levin [38]). Furthermore, the spatial modeling of energy commodity markets has often involved various extensions beyond the basic SPE approach (e.g., Kennedy [32], Yang and Labys [77] and Nagurney [52]). These extensions include linear complementarity programming, entropy maximization, or network flow models. For the detailed description of the advances in the spatial equilibrium models, readers are referred to Labys and Yang [35]. Computational algorithm was developed by Nagurney [51]; applications and statistical sensitivity analysis by Yang and Labys [76]; mathematical sensitivity analysis by Irwin and Yang ([27], [28]); spatial equilibrium model with transshipment by Tobin and Friesz [67]; applications of linear complementarity problem by Takayama and Uri [65] and

Yang and Labys [77], Beyond that imperfect spatial competitions include works by Yang [73]; variational inequality by Dafermos and Nagurney [7], iterative approach by Nagurney [51]; dispersed spatial equilibrium model by Harker [21]; spatial diffusion model by Yang [74]; spatial pricing in oligopolistic competition by Sheppard et al. [60]; and the spatial tax incidence by Yang and Page [78]. The advances and applications of the spatial equilibrium model can be found in Labys and Yang ([35] and [36]).

In particular, the spatial price equilibrium (SPE) has much richer policy implementations since each region has a price sensitive demand and a supply function. The linear programming transportation (LPT) model, on the other hand, has fixed demand and capacity constraints and as such lacks policy implications (Henderson [23]). The SPE model, in contrast, has been widely implemented both in theoretical advances and in empirical applications. Applications of SPE models include a wide range of agricultural, energy and mineral commodity markets as well as international trade and other spatial problems, readers are referred to the works by Labys and Yang [35]. Advances in computer architecture (parallel processing) have led to large scale computations in spatial equilibrium commodity and network models (Nagurney [51], Nagurney et al. [53]) and in spatial oligopolistic market problems (Nagurney [52]), spatial Cournot competition model by Yang et al. [75]. Most recent application on SPE can be found in lumber trade in North America by Stennes and Wilson [63]. In addition, the SPE model has become increasingly fused with other types of spatial models. For instance, the solutions of a SPE model can be obtained and combined with the gravity model (Harker [21]) and the commodity or passenger flows can also be estimated using econometric (e.g., the logit model, Levin [38]). Furthermore, the spatical modeling of energy commodity markets has often involved various extensions beyond the basic SPE approach (e.g., Kennedy [32], Yang and Labys [76] and Nagurney [52]). These extensions include linear complementarity programming, entropy maximization, or network flow models. It is interesting to note that the large-scale

Leontief-Strout was published one year before Takayama and Judge reformulated the Enke-Samuelson problem into a standard quadratic programming or spatial equilibrium model. The entropy modeling had not received enough attention until 1970 when Wilson derived the gravity model from the entropy-maximizing paradigm. By the middle of the 1970's Wilson and Senior [72] proved the relationship between the linear programming and the entropy-maximizing model[5]. As a matter of fact, Hitchcock-Kantorovich-Koopmans linear programming transportation problem was shown to be a special case of the entropy model. The detailed descriptions on these models may be found in Batten and Boyce [2] and the combinatorial calculus by Lewis and Papadimitriou [39]. However, the implementation of such entropy models to the interregional commodity shipment problem has been limited despite the recent result by Yang [74].

Extensions on the entropy maximization model include the following. Sakawa et al. [58] employed a fuzzy programming to minimize production and transportation costs when demand estimation and capacities of the factories were not precise. The application of fuzzy set theory started with Zadeh [80]. Facility-location problem was solved using decomposition algorithm by Beasley [3]. Based on the Shannon entropy, Islam and Roy [29] reduced a multi-objective entropy transportation model with the trapezoidal costs to a geometric programming problem. Liu and Kao [45] applied and solved a fuzzy transportation problem with linear membership function. Verma et al. [69] solved a multiple objective transportation problem with nonlinear function. Li [40] adopted a Markov chain model to improve on the estimates on the origin-destination trip matrix derived from the entropy model by Van Zuylen and Willumsen [68]. According to Li [40], large-scale direct sampling using statistical inference for origin-destination table (Li and Cassidy [43]) may not be efficient. Another approach -balancing method- by Lamond and Stewart [37] may fail to converge if the original estimated trip matrix contains too many zeros and if an entry in a

[5] An alternative formulation was established by Erlander ([12], [13]).

reference matrix is zero, this entry retain a zero in every iteration (Ben-Akiva et al. [4]). In addition, Hazelton [22] utilized a Bayesian analysis to integrate prior information with current observations on traffic flow. Li and Moor [41] tackled the estimation problem in a dynamic setting and in the presence of incomplete information. Most recently, Giallombardo et al. [16] integrate the berth allocation of incoming ships with quay crane assignment problem: number of quay cranes per working shift. To realize economies of scale, shippers build larger containers and demand ports to have enough facility to handle them. The service allocation problem was formulated as generalized quadratic assignment problem (Hahn et al. [18]), which might well require integer solution. Other approaches include column generation heuristic for a dynamic generalized assignment problem (Moccia et al. [48]), a Lagrange multiplier approach (Monaco and Sammarra [49]), a genetic algorithms (Nishimura et al. [54]) and a stochastic beam search approach (Wang and Lim [70]).

The linear programming transportation model is a special case of the entropy maximization model. To a large extent, the entropy model was developed by Wilson [71]. The linear programming transportation model is limited in producing numbers of positive-valued solution depending on the number of independent constraints. As such if one requires the solution value to be integer, this weakness may disappear. From another perspective, the entropy model produces many positive-valued variable solutions, which fits the concept of market diffusion or maximum market penetration.

The classical transportation model dates back to Hitchcock [24], Kantorovich [30] and Koopmans [33] with a variety of efficient algorithms developed by Dantzig [9] and Russell [56]. Applications of the model are primarily on agricultural, mineral and energy markets including works by Henderson [23], Devanney III and Kennedy [10], the Project Independence Evaluation System by Hogan and Weyant [25], and lead and zinc markets by Dammert and Chhabra [8].

The linear programming transportation (LPT) problem evolved into the quadratic programming transportation (QPT) models by Samuelson [59], and Takayama and Judge [64], which relaxed the assumption on fixed demand and supply requirements to quadratic programming models. Applications and extensions of the QPT model proliferated starting from the late 1970's: see Labys and Yang [34], and Hwang et al. [26]. The QPT models become a special case of the linear complementarity programming (LCP) model by Karamardin [31]. The transportation models in terms of LCP include works by Glassey [17], Irwin and Yang ([27], [28]) among others. Floran and Los [14] formulate the LCP version of the transportation problem into a variational inequality problem which leads to various papers in the field including Harker [19] and Nagurney [52]. A survey on the transportation problem can be found in Labys and Yang ([35] and [36]).

Despite the advances in the classical transportation model, a nonlinear integer programming transportation (NIPT) model has not advanced much in the literature. The purpose of this paper is to propose possible solutions to NIPT problems with a variety of formulations. The next section presents different transportation models. Section 3 proposes straightforward solutions via a Visual Basic program with an Excel based user interface. For the remainder of this paper, Excel denotes a Microsoft Excel spreadsheet which is the user interface to a Visual Basic program. Section 4 presents a discussion of the combinatorial aspects of the problem. Finally, Section 5 provides suggestions and conclusions.

2 Model Formulation

Well known in the literature, a typical linear programming transportation (LPT) problem takes the following form:

Minimize:

$$x_{ij \in R_+}$$

$$TC = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \tag{1}$$

subject to:

$$\sum_{i \in I} x_{ij} \geq D_j \quad \forall \quad j \in J \tag{2}$$

$$\sum_{i \in I} x_{ij} \leq K_j \quad \forall \quad i \in I \tag{3}$$

$$x_{ij} \geq 0 \quad \forall \quad ij \in (I \times J) \tag{4}$$

where

x_{ij} = volume of shipment from supply source *i* to demand sink *j*

c_{ij} = unit transportation cost from supply source *i* to demand region *j*

D_j = fixed level of consumption in demand sink j

K_i = fixed level of production capacity in supply source *i*

I, *J* are positive integer sets of (1,…, *m*) and (1,…, *n*)

$I \times J$ is the Cartesian product of *I* and *J*

R_+ represents a set of all the nonnegative real numbers.

The solution and model formulation are well known (e.g., Gass [15]) and applications are abundant. The application of linear integer programming (LIP) models became popular with the advent of the branch and bound algorithm coupled with software like LINDO (Yang and Pineno [79] and Brusco and Johns [5]). However, the LIP formulations are restricted to linear objective functions, which are less general than nonlinear versions.

Since the seminal paper by Murtagh and Saunders [50], solutions to large-scale, nonlinear programming (NLP) problems (linear constraints) became plausible with software such as MINOS producing efficient solutions to well-conditioned problems. On the other hand, nonlinear integer programming (NLIP) problems have just begun to receive attention (Bussieck and Pruessner [6], Li and Sun [42]). In late 1980s, Mawengkang and Murtagh [46] solved a quadratic assignment problem using Murtagh's direct search procedure while Ravindran et al. [55] solved a 3-variable and 3-constraint quadratic integer problem. Despite the emerging applications in finance, engineering, management science and operations research, NLIP has not advanced as much as LP due to the combinatorial nature of the integer requirements and nonlinearity of the objective function. Thus, it is not surprising that solving NLI problems remains challenging and that solution methodologies for large-scale NLI problem are in the experimental stage.

According to Bussieck and Pruessner [6], the four major methods-Outer Approximations, Branch and Bound, Extended Cutting Plane and Generalized Bender's Decomposition-guarantee global optimality under the assumption of generalized convexity. In general, convexity in the original problem coupled with a branch-and-bound technique is required for obtaining solutions. Other methods such as open algorithm allow for choice of methods to solve a particular problem only for a skilled user. In addition, experimental results indicate that a problem of several hundred integer variables can be solved using the concept of the filled function (Li and Sun [42]).

As NLIP starts to make important progress, two problems remain. First, if the original NLP has a convex objective function, good NLP software with the branch-and-bound technique is generally sufficient to produce a global minimum. If, however, the objective function is not convex in a minimization problem, the solution may well be only locally optimal as is the case for a nonlinear transportation problem. Second, many users in business management are not mathematically skilled enough to grasp concepts such as outer approximation, generalized reduce gradient, convex relation, convexification and filled function. As such, we propose an alternative approach to solving the NLIP model for a small or medium sized problem using only EXCEL. Our approach can produce global optimality even for a non-convex objective function for a small or medium-sized problem. Furthermore, the result is easy to comprehend without heavy mathematics.

3 Excel-based Solutions

The LPT problem (equations 1 through 4) can easily be converted into an NLIP if the x_{ij}'s are integer-valued and the unit transportation cost function c_{ij} is no longer a constant. To illustrate our approach, we construct a transportation problem of 5 supply sources (i = 1,…,5) and 3 demand sinks (i = 1, 2, 3) with a total of 15 possible shipments. The solution to the linear integer transportation problem (three by five) using LINDO is reported in Table 1.

Table 1: Optimum solution of the linear integer programming transportation model

Demand Sink / Supply Source	1	2	3	Total
1			10	10
2		2	10	12
3		15		15
4	9	9		18
5	16			16
Total	25	26	20	71

a: Objective function value = 609.7
b: Solution time = 15.93 minutes with EXCEL
c: Unit transportation cost from source *i* to sink *j* for x_{11}, x_{12},…, x_{53} are 9.5, 11, 8, 12, 10.4, 10, 12, 7.5, 14, 10, 9.6, 11.2, 7.5, 8, 9.1 respectively.

Notice that assumed parameters bear no empirical relevance. Consistent with the result from LP, the number of positive-valued variables (7) cannot exceed the number of independent constraints (5 + 3 - 1 = 7). In the case where one has less than 7 positive-valued shipments, the transportation system is known as “degenerate,” in which the transportation network can be decomposed into two or more independent systems (Gass [15]).

The next case involves declining unit transportation cost as the volume of units shipped between a source and a destination increases. This rate break model is common in business practices where the cost of a shipment between a source and a destination is insensitive to the quantity shipped. Thus in this type of situation, the larger the number of units shipped, the lower the cost per unit shipped. As such for each pair of *i* and *j*, we have following separable unit transportation cost:

$$c_{ij} = b_{ij} - a_{ij} x_{ij} \tag{5}$$

where: $b_{ij} > 0$, $a_{ij} > 0$ and $b_{ij} > a_{ij}x_{ij}$ to ensure positive c_{ij}. Note that the unit cost function need not be separable: c_{ij} can be a function of x_{ij} and x_{rs} for $r \neq s$ and $s \neq j$. The separability of the cost function is assumed for simplicity and thus can be easily expanded. Therefore, for a given set of a_{ij} and b_{ij} (Table 2A), the LPT model becomes a concave quadratic integer programming transportation (QIPT) problem for which the solution is presented in Table 2.

An examination of Table 2 reveals immediately that there exist 6 positive shipments in the solution set. The lack of shipments is not unexpected as least cost routes within the constraints must be heavily used to minimize the total transportation cost. It is interesting to note that the least-cost solution with the objective function value 799.6 (*Z* = 799.6) is far superior to the second best solution produced by Excel, which has 8 positive shipments (*Z* = 831.8) or the third best solution with 7 shipments (*Z* = 832.7). Excel can easily record and display a large number of solutions which is a strong point of our methodology as users may be interested in comparing non-optimal solutions to the optimal solution.

As Excel obtains each feasible solution, it calculates the objective function value of that solution and then stores that solution and its objective function value in a file with previously found solutions sorted on solution value. In this study we recorded the best three solutions found which entailed comparing the current solution found to the best three already found. If the current solution is better than any of the three best solutions found so far, the worst of the three was dropped and the current solution was saved.

Table 2: Optimum solution of the quadratic integer programming transportation model - declining unit transportation cost (linear)

Demand Sink / Supply Source	1	2	3	Total
1		10		10
2	12			12
3	13		2	15
4			18	18
5		16		16
Total	25	26	20	71

a: Objective function value = 799.6
b: Solution time = 23.77 minutes with EXCEL
c: Unit transportation cost function takes the form of $b_{ij} - a_{ij}x_{ij}$ for every *i*th source and *j*th sink (see Table 2A).

Table 2A: Unit transportation cost (t_{ij}) - coefficients $c_{ij} = b_{ij} - a_{ij}x_{ij}$

Demand Sink / Supply Source			
a_{ij}	1	2	3
1	1	1.5	1.6
2	2	1.1	1.2
3	0.6	0.7	1
4	0.5	0.9	1.1
5	1	1.6	1
b_{ij}	1	2	3
1	30	27	28.5
2	36	28.8	39
3	24	27	25.5
4	23.4	33	30
5	36	31.5	33

Because of our solution methodology we are not hampered in getting global optimality because of the wrong curvature of the objective function. As long as the function is real-valued, be it concave or convex, separable or nonseparable, smooth or discrete, Excel can produce a set of optimum solutions.

We now proceed to another case in which unit transportation cost declines exponentially in the form of

$$c_{ij} = e^{-f_{ij}(x_{ij}-g_{ij})} \tag{6}$$

where $f_{ij} > 0$ is a parameter and $g_{ij} > 0$ represents a threshold shipment level beyond which rate discount starts to apply. Unit costs decline at decreasing rates as compared with the previous case in which unit cost decreases linearly (constant rate).

Table 3: Optimum solution of the exponential integer programming transportation model - declining unit transportation cost (exponential) without minimum threshold shipments

Demand Sink / Supply Source	1	2	3	Total
1	10			10
2			12	12
3	15			15
4		10	8	18
5		16		16
Total	25	26	20	71

a: Objective function value = 0.1478
b: Solution time = 34.18 minutes with EXCEL

Applying the same procedure via Excel, the optimum solution is reported in Table 3 with the assumed values of the parameters in Table 3A. Notice that, like in the previous case, the optimum solution consists of only 6 positive shipments indicating heavy usage of the least cost routes to minimize the total transportation cost. This lack of positive shipments leaves some routes little used or unused and other routes overused an often undesirable phenomenon for resource usage. To ensure enough commodity shipment for each route, we force all shipments to be greater than or equal to the threshold level, i.e., $x_{ij} > g_{ij}$.

The results are shown in Table 4 using the cost parameters and as is readily evident, shows a radically different set of optimum solutions. All of the shipments are positive signifying the spatially diffusing transportation network. The best three solutions from Excel have similar objective function values ranging from 19.67 to 19.80. It should also be noted that as more requirements are introduced into the model, and thus the number of feasible solutions are reduced, the solution time[6] for Excel is greatly reduced, i.e. from 34.18 minutes in Table 3 to ≈ 1 second in Table 4.

When the unit transportation cost increases linearly reflecting a tight supply condition for some routes, the unit cost takes the form of the following:

$$c_{ij} = p_{ij} + q_{ij} x_{ij} \tag{7}$$

where $p_{ij} > 0$ and $q_{ij} > 0$. For simplicity, we employ the same cost coefficient in the case of linear rate break: $p_{ij} = b_{ij}$ and $q_{ij} = a_{ij}$. Since the unit costs get more expensive as the size of a particular shipment increases, shippers tend to diversify their cargo among different routes as much as possible.

The results shown in Table 5 indicate that all of the 15 shipment routes are used. The solution is vastly different when compared with the solution of the ILP in which there are only 7 positive shipments. The convex objective function in this case is mathematically "appropriate" for a minimization problem and the integer requirement does not alter the solution property. The solution to the convex quadratic transportation problem generated by Excel can also be derived by a NLP

[6] All computations for this paper were done a PC with a 3.0 GHz Pentium 4 processor with 512 MB of memory.

software on continuous variables (e.g., MINOS or LINGO) as the original problem.

Table 3A: Unit transportation cost (t_{ij}) - coefficients $C_{ij} = e^{[-f_{ij}(x_{ij}-g_{ij})]}$

Demand Sink / Supply Source			
f_{ij}	1	2	3
1	1	0.15	1.6
2	2	1.1	1.2
3	0.6	0.7	1
4	0.5	0.9	1.1
5	1	1.6	1
g_{ij}	1	2	3
1	4	2	3
2	5	2	2
3	2	2	3
4	2	3	2
5	5	3	4

a: f_{ij} = estimated parameter of the exponentially declining unit transportation cost

b: g_{ij} = minimum shipment required for the rate break

With the solution x_{11} = 2.726, x_{12} = 4.187, x_{13}= 3.087,…, we may perform the branch-and-bound technique as follows. First branch on x_{11}, with $x_{11} \geq 3$ versus $x_{11} \leq 2$ indicates that $x_{11} \geq 3$ (or $x_{11} = 3$) has lower objective function value (Z_1 = 2433.271 < Z_2 = 2434.224) and hence we have x_{11} = 3 (or subproblem #1).

Branching further, on x_{12} based on subproblem #1 with $x_{12} \leq 4$ versus $x_{12} \geq 5$ indicates $x_{12} \leq 4$ (or x_{12} = 4) has lower objective function value (Z_3 = 2433.279 < Z_4 = 2436.573). The similar quadratic branch-and-bound procedures can go on until all optimum x_{ij}’s are integer-valued.

Table 4: Optimum solution of the exponential integer programming transportation model - declining unit transportation cost (exponential) with minimum threshold shipments

Demand Sink / Supply Source	1	2	3	Total
1	6	2	2	10
2	6	3	3	12
3	5	7	3	15
4	2	10	6	18
5	6	4	6	16
Total	25	26	20	71

a: Objective function value = 19.68
b: Solution time $\approx$ 1 second with EXCEL
c: Unit transportation cost c_{ij} declines exponentially with increase in shipment: $c_{ij} = e^{[-f_{ij}(x_{ij}-g_{ij})]}$ where g_{ij} is the minimum shipment required for the rate break. See Table 3A for parameter values.

Obviously, the process can be tedious, but it converges to global optimality if the objective function is convex as is the case of increasing unit transportation cost. If, however, the problem is ill-conditioned with a great deal of nonlinearity as in the case of exponentially declining unit cost, the convergence may be lethargic. As a result, the approach from using Excel may actually be preferred for small or medium-sized problems.

The computation times are quite reasonable: 23.77 minutes for the ill-conditioned or concave programming problem (Table 2), 34.18 minutes for the ill-conditioned exponentially declining unit cost problem (Table 3), approximately one second for the exponentially declining unit cost problem with threshold shipments for rate break (Table 4), and 23.72 minutes for the convex quadratic (increasing unit cost) problem (Table 5).

Table 5: Optimum solution of the quadratic integer programming transportation model - linearly increasing unit transportation cost

Demand Sink / Supply Source	1	2	3	Total
1	3	4	3	10
2	2	8	2	12
3	5	6	4	15
4	11	3	4	18
5	4	5	7	16
Total	25	26	20	71

a: As those in Table 2A with $p_{ij} = b_{ij}$ and $q_{ij} = a_{ij}$

b: Objective function value = 2434.8

c: Solution time = 23.72 minutes with EXCEL

d: Unit transportation cost t_{ij} increase with the shipment x_{ij} :

$c_{ij} = p_{ij} + q_{ij}x_{ij}$ reflecting a tight supply condition. We use the same coefficients as those in Table 2A with $p_{ij} = b_{ij}$ and $q_{ij} = a_{ij}$.

It comes as no surprise that both quadratic integer programming problems take about the same amount of computation time (Tables 2 and 5). Although Excel does not distinguish convexity from concavity of objective functions, additional information on x_{ij} 's, be it upper bound or lower bound, can reduce computation time precipitously even in the case of the very nonlinear exponential cost curve.

4 Combinatorial Analysis of the Transportation Problems

The Excel-based approach is easy to implement for virtually all kinds of objective functions. No advanced mathematical training in optimization is required, nor is software knowledge on NLP. For the balanced transportation problems, the special feature in the constraint sets can be exploited to reduce the number of possible integer values. For instance, the constraint that $x_{11} + x_{12} + x_{13} =$ 10 gives rise to no more than 11 = 10 + 1 by 11 = 10 + 1 or 121 possible integer combinations, since the last position is automatically implied by the constraint. However, not all of the 121 integer combinations are feasible. That is, if $x_{11} = 10$, x_{12} cannot assume any other values except 0. Thus, 10 integer combinations are eliminated from the total of 121. If $x_{11} = 9$, 9 integer combinations are eliminated for x_{12} can assume the value of either 0 or 1. If $x_{11} = 8$, 8 integer combinations are eliminated and so on. The number of feasible possible integer solutions reduces to 121 - (1 + 2 + … + 10) = 66. The answer is exactly consistent with the well-known combinatorial formula (Lewis and Papadimitriou, 1981) in calculating ways in assigning *b* number of balls (right-hand-side constraint) into *u* number of urns (number of variables).

$$ {}_{b+u-1}C_{u-1} = {}_{10+3-1}C_{3-1} = 66 \tag{8} $$

By the same token, there exist ${}_{14}C_2$ = 91 for the second constraints: $x_{21} + x_{22} + x_{23} = 12$.

On the demand side, there exist ${}_{25+2-1}C_{5-1}$ = 14950 integer combinations for the first constraint $x_{11} + x_{21} + \cdots + x_{51} = 25$. The intersection of demand and supply constraints reduces the feasible integer solutions to a reasonable number before substituting them into the objective function. It is to be pointed out that increasing the number of regions carries larger computation cost than increasing the right-hand-side constraints. For instance, doubling the right-hand-side constraint $K_1 =$ 10 to K_1 = 20 increases possible integer combinations from 66 to ${}_{22}C_2$ = 232. However, doubling the supply region (*j* = 3 to *j* = 6) with K_1 = 10 unchanged leads to ${}_{10+6-1}C_{6-1}$ = 3003 possible integer combinations, quite a bit greater increase than that from doubling K_1.

5 Concluding

The combinatorial nature of integer programming is inevitable even after taking specific model structure into consideration. This is the root problem in implementing large-scale nonlinear integer programming models regardless of which algorithm one chooses to use. Consequently, we suggest that the size of origin-destination be moderate. In the case of large origin-destination problems, more information on the size of x_{ij} is needed to drastically reduce the dimensionality problem. For instance, if x_{ij} is to be greater than the threshold value to be eligible for the rate break (Table 4), computation time can be noticeably reduced.

In the case of huge right-hand-side constraints, we suggest scaling these values to the nearest thousands or millions. The approach from Excel proposed in this paper is particularly appropriate if one can balance the sizes of origin-destination and right-hand-side constraints in such a way that computation time is not excessive. For a large-scale problem, one must exploit the structure of the model and acquire more information on the bounds of discrete variables. Our approach certainly provides an alternative way to solve nonlinear integer programming models with virtually all kinds of algebraic functions even for laymen who do not feel comfortable with mathematic programming jargons.

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