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Residual Grey Markov Model for the developments of GDP, Population and Energy Consumption in Iran

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Abstract

Grey system theory can powerfully deal with incomplete and uncertain information. In this paper, we introduced an improved grey GM(1,1) model that integrates residual modification with Markov chain model. By this model, we improved the forecast accuracy of original grey forecast model and achieved the predictions and analyses of GDP, Population and Energy consumption of Iran from 2009 to 2021 Based on the data from 1992 to 2006. We examined development relationship between GDP, Population and Energy consumption in the future.

JEL classification numbers: C15, C53, C63

Keywords: Grey theory, Markov chain, Prediction, Energy Consumption

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1 Introduction

Energy Consumption forecasting is an essential factor in setting up energy plans. The optimal energy policies and planning energy formation can be made through energy assumption forecasting. Energy resources in Iran consist of the third largest oil reserves and the second largest natural gas reserves in the world. Natural gas and oil consumption both account for about half of Iran's domestic Energy consumption.

In 1982, Deng proposed grey system theory [1] to study the uncertainty of systems. The grey system theory can perform grey relational analysis for sequences and is mainly utilized to study uncertainties in system models, analyze relations between systems, establish models, and make forecasts and decisions. X. Wang [9], considered the grey models as dynamic models by dealing with great value, which not only give middle information for establishing a model, but reduce the randomness, irregularity and interference of the original data, making the long-term trend more close to the fact.

Grey prediction theory wants to find the optimized system parameters for a grey differential equation so that the dynamic behavior of the system can be best fitted with the differential equation. Some traditional forecasting models, such as regression models and artificial neural network-based models are frequently adopted in many fields. However, their predictive accuracy is low when the sample is small. The advantages of grey model are that it can be used in circumstances with less data. The fields covered by the grey system theory include system analysis, data processing, modeling, prediction, decision- making and control. For achieving higher forecasting precision, researchers begin to shift their attention to improve the original GM(1,1). G. Li and et al [5], applied the residual error modification with Markov-chain sign estimation for the developments of GDP, Population and Energy consumption of China.

C. Sun and G. Lin [8], applied hybrid Grey Forecasting model for Taiwan's Hsinchu science industrial park. C. Shih and et al [7], proposed a novel grey modification model with progression technique which can produce grey number prediction and deal with the irregular time series more flexibly. C. Hsu and C. Chen [3], proposed a modified GM(1,1) model and a modification method by using the economic growth information to modify the model coefficient for long-term forecasting. Y. Zhang and et al [10], combined grey model with partial least squares regression and applied the method to forecast city terminal energy consumption of Dalian. X. Liu and et al [6], proposed an improved GM(1,1) and applied it to Chinese population problem. Sh. Kordnoori and H. Mostafaei [4], applied the Grey Markov Model for predicting the crude oil production and export of Iran.

As shown in Figure 1, GDP, Population and Energy consumption are related to each other. In this paper, we proposed hybrid grey-based model to improve the Grey model precision and applied our model to predict the GDP, Population and Energy consumption of Iran from year 2009 to 2021 and finally analyze their developments and growth .

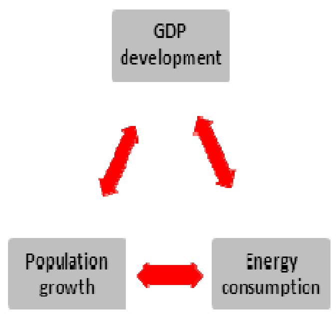

Figure 1: The relationship between GDP, Population and Energy consumption

2 The Proposed Methodology

This section introduces how to establish the mathematical model.

2.1 To establish GM (1, 1)

Denote the original data sequence by $X^{(0)}(k)=\left\{x^0(1),x^0(2),...,x^0(n)\right\}$

The Accumulated Generation Operation (AGO) is expressed as

$$X^{(1)}(k)=\sum_{n=1}^{k}x^0(n) \tag{1}$$

The first – order differential equation of GM (1, 1) model is then give as

$$\frac{dX^{(1)}}{dT}+aX^{(1)}=b \tag{2}$$

The solution for (2) is

$$\hat{x}^{(1)}(k)=(x^{(0)}(1)-\frac{b}{a})\cdot(1-e^{a})\cdot e^{-a(k-1)}, \qquad k=2,3,...,n \tag{3}$$

where $x^{(1)}=x^{(0)}(1)$ and the coefficients a and b are called developing and grey input coefficient, respectively. By least-square method, they can be obtained as

$$\begin{pmatrix} a \\ b \end{pmatrix}=(B^TB)^{-1}B^TY_n \tag{4}$$

where

$$B=\begin{pmatrix} -\frac{1}{2}(x^{(1)}(1)+x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2)+x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1)+x^{(1)}(n)) & 1 \end{pmatrix} \tag{5}$$

$$Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{6}$$

2.2 Markov Residual Modified Grey Model (MRMGM)

Residual errors of Grey Model are obtained using Markov Chain. Markov Chain predicts the future development according to the transition probability among states, which reflects the internal law of all states. Therefore, markov method can be used for predicting of the system with high fluctuation. Define residual series $e^{(0)}$ as

$$e^{(0)} = \left[e^{(0)}(2),\ e^{(0)}(3),\ ...,\ e^{(0)}(n) \right] \tag{7}$$

where $e^{(0)}(k) = x^{(0)}(k) - \hat{x}^{(0)}(k)$, $k = 2,3,\ldots,n$.

Denote absolute values of residual series as $\varepsilon^{(0)}$

$$\varepsilon^{(0)} = \left[\varepsilon^{(0)}(2), \varepsilon^{(0)}(3), ..., \varepsilon^{(0)}(n) \right] \tag{8}$$

where $\varepsilon^{(0)}(k) = \left| e^{(0)}(k) \right|$, $k = 2,3,\ldots,n$.

A GM (1, 1) model of $\varepsilon^{(0)}$ can be established as

$$\hat{\varepsilon}^{(k)} = (\hat{\varepsilon}^{(0)}(1) - \frac{b_\varepsilon}{a_\varepsilon}) \cdot (1 - e^{a_\varepsilon}) \cdot e^{-a_\varepsilon (k-1)} \tag{9}$$

where a_τ, b_τ are estimated using OLS.

Assume that sign of k*th* data residual is in state 1 when it is positive and in state2 when it is negative. A one step transition probability P is associated with each possible transition from state *i* to state *j*, and P can be estimated using

$$P_{ij} = M_{ij} / M, \qquad i, j = 1, 2,$$

M_i means the number of years whose residuals are state *i*, and M_{ij} is number of transitions from state *i* to state *j* that have occurred.

These P_{ij} values can be denoted as a transition matrix *R*:

$$R = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{10}$$

Denote the initial state distribution by the vector $\pi^{(0)}=\left[\pi_1^{(0)} \quad \pi_2^{(0)}\right]$, where $\pi_1^{(0)}$ and $\pi_2^{(0)}$ are the transition possibility of state1 and state2. Set the n*th* data to be the initial state, and state transition possibility vector to be $\pi_1^{(0)}$.

Calculation of the state possibility vector of (i+1)*th* step transformation after initial state is as follow:

$$\pi^{(\iota+1)}=\pi^{(0)}R^{(i+1)} \tag{11}$$

where $\pi^{(\iota)}$ are k *th* step residual state probabilities.

Let the sign of the k step residual be represented as follows:

$$\delta(i+1)=\begin{cases}+1, & \text{if} \quad \pi_1^{(\iota+1)}>\pi_2^{(\iota+1)} \\ -1, & \text{if} \quad \pi_1^{(\iota+1)}<\pi_2^{(\iota+1)}\end{cases} \tag{12}$$

An improved Grey model with residual modification and Markov Chain sign estimation can be formulated as

$$\hat{x}_r^{(0)}(k)=\hat{x}_0^{(1)}(k)+\delta(k)\left[\hat{\varepsilon}^{(0)}(1)-\frac{b_\varepsilon}{a_\varepsilon}\right]\cdot(1-e^{a_\varepsilon})\cdot e^{-a_\varepsilon(k-1)} \tag{13}$$

where $\hat{x}_r^{(0)}=x^{(0)}(1)$ and $\delta(k)=\pm 1$.

2.3 Error Analysis

To examine the accuracy of forecasting models, comparison of forecasting results can be calculated. Relative Percentage Error (RPE) compares real and forecast values as

$$RPE=\frac{\left|x^{(0)}(k)-\hat{x}^{(0)}(k)\right|}{x^{(0)}(k)}\times 100\% \tag{14}$$

3 Main Results

The GM(1,1) provides an excellent approach to the development dynamic system consist of population, GDP and energy consumption. Deng [2], considered the energy system which is an imperfect information system as a grey one. Iran, a member of the OPEC (Organization of Petroleum Exporting Countries), ranks among the world’s top three holders of both proven petroleum reserves. Natural gas accounts for half of Iran’s total domestic energy consumption, while the remaining half is predominantely oil consumption. The population of Iran is increased dramatically since 1978 (Figure 2), so it is essential to predict it in future. In this study, we used the data of GDP, Population and total energy consumption of Iran from 1992 to 2006 [11] , [12], as the model– fitting (Table 1) and the data for 2007 and 2008 are utilized as expose testing.

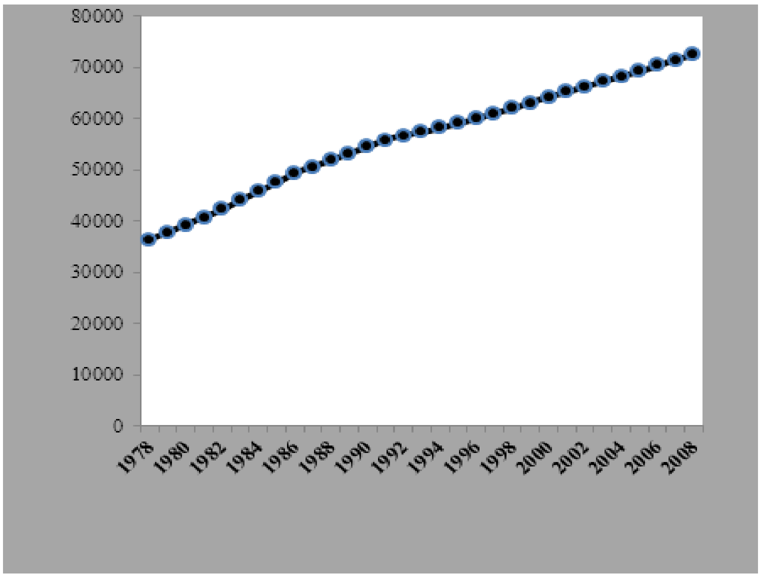

Figure 2: The trend of population of Iran from 1978

Table 1: Data of Population, GDP, and Energy consumption of Iran (1992-2006)

Year	Population	Energy consumption	GDP
1992	56656.0	496.2	254822.5
1993	57488.0	512.3	258601.4
1994	58331.0	559.6	259876.3
1995	59187.0	561.6	267534.2
1996	60055.0	625.2	283806.6
1997	61070.0	658.0	291768.7
1998	62103.0	680.9	300139.6
1999	63152.0	695.2	304941.2
2000	64219.0	683.2	320068.9
2001	65301.0	682.8	330564.8
2002	66300.0	730.7	357670.9
2003	67315.0	768.4	385630.3
2004	68345.0	831.0	410428.8
2005	69390.0	903.2	438899.9
2006	70496.0	998.9	467930.0

Now we are applying the proposed model to forecast the population, GDP and total energy consumption. From the data in table 1 and the GM (1, 1) model, we obtain:

$$\hat{x}^{(0)}(k) = 56485.751 \cdot e^{0.015892(k-1)} \quad \text{(Population)}$$

$$\hat{x}^{(0)}(k) = 227095.0723 \cdot e^{0.048677(k-1)} \quad \text{(GDP)}$$

$$\hat{x}^{(0)}(k) = 497.773 \cdot e^{0.044378(k-1)} \quad \text{(Energy consumption)}$$

The absolute values of residual series are:

$$\varepsilon^{(0)} = (97.40645, 21.06943, 56.99440, 138.0209, 87.24991, 33.92480, 19.70694, 75.39393, 129.8808, 84.90789, 39.21168, 8.47578, 58.42665, 64.91748) \quad \text{(Population)}$$

$$\varepsilon^{(0)} = (20178.56, 9560.643, 4732.501, 7896.041, 2095.396, 3982.952, 14351.34, 15150.33, 21375.57, 11824.67, 2296.146, 3152.123, 11307.78, 19009.08) \quad \text{(GDP)}$$

$$\varepsilon^{(0)} = (8.06072, 15.62665, 7.05747, 30.73832, 36.56317, 31.26396, 16.08515, 26.73134, 59.34620, 45.12289, 42.62774, 16.83010, 16.89755, 72.37941) \quad \text{(Energy Consumption)}$$

The Markov Residual Modified Grey Models (MRMGM) is:

$$\hat{x}_r^{(0)}(k) = 56485.751 \cdot e^{0.015892(k-1)} + \delta(k)(70.596 \cdot e^{-0.013883(k-1)}),\ \ k = 2,3,\ldots$$

(Population)

$$\hat{x}_r^{(0)}(k) = 227095.0723 \cdot e^{0.048677(k-1)} + \delta(k)(5739.885 \cdot e^{0.050607(k-1)}),\ \ k = 2,3,\ldots$$

(GDP)

$$\hat{x}_r^{(0)}(k) = 497.7773 \cdot e^{0.044378(k-1)} + \delta(k)(29.058 \cdot e^{0.008114(k-1)}),\ \ k = 2,3,\ldots$$

(Energy Consumption)

As comparing with GM (1, 1) model, the forecast values of 2007 and 2008 by these two methods are listed in Table 2.

Table 2: The comparison between GM (1, 1) and MRMGM forecasting values.

	Years	Actual value	GM(1,1) model forecast precision value	MRMGM Forecast precision value
Population	2007	71532	71691.23 99.78	71633.82 99.85
	2008	72584	72839.65 99.65	72783.03 99.73
GDP	2007	499071.1	471313.63 94.44	483576.11 96.89
	2008	501000.0	494823.31 98.77	507701.71 99.99
Energy consumption	2007	1084.0	968.56 89.35	1001.382 92.38
	2008	1115.1	1012.601 90.80	1045.601 93.77

The Table 2 shows a better precision obtained by the markov residual modified grey model, therefore we predict the population, GDP and energy consumption and supply from 2009 to 2021 by MRMGM(1,1). The forecasted values are listed in Table 3. Moreover, we analyze the development of population, GDP and energy consumption (Figure 3) and to analyze their relationship more, we investigate the correlative comparision values of Energy/GDP, Energy/Population and GDP/Population from 2009 to 2021 (Figure 4).

Table 3: The predicted values by MRMGM

Year	Population	GDP	Total energy consumption	Year	Population	GDP	Total energy consumption
2009	73950.62	533074.3	1091.815	2017	83989.61		1595.184
2010	75136.89	559692.2	1140.117	2018	85336.52	787192.4	1613.975
2011	76342.14	587639.2	1190.601	2019	86704.99	826499.8	1685.877
2012	77566.67	616981.8	1243.365	2020	88095.36	867770.0	1761.031
2013	78810.80	647789.6	1298.513	2021	89507.98	911101.0	1839.585
2014	80074.84	680135.9	1365.154				
2015	81359.10	714097.3	1416.900			956595.8	
2016	82663.92	749754.6	1479.368				

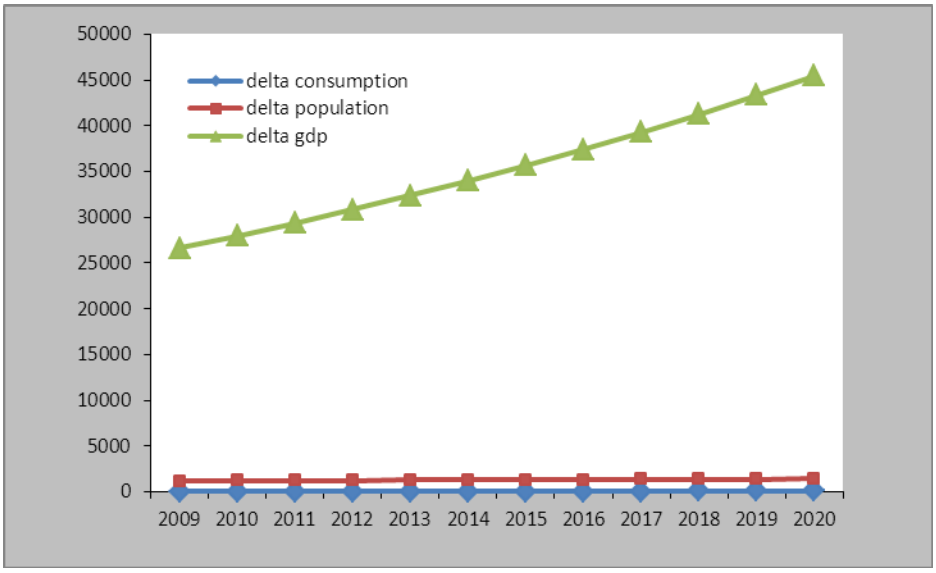

Figure 3: The development trends of population, GDP, energy consumption from 2009 to 2020

We realize that ΔGDP> ΔPopulation >ΔEnergy consumption from 2009 to 2021. Moreover, along with the development of GDP and Population, the energy will occur insufficiently. We know that Energy/GDP will decrease slowly, Energy/Population will increase slowly and GDP/Population will increase quickly.

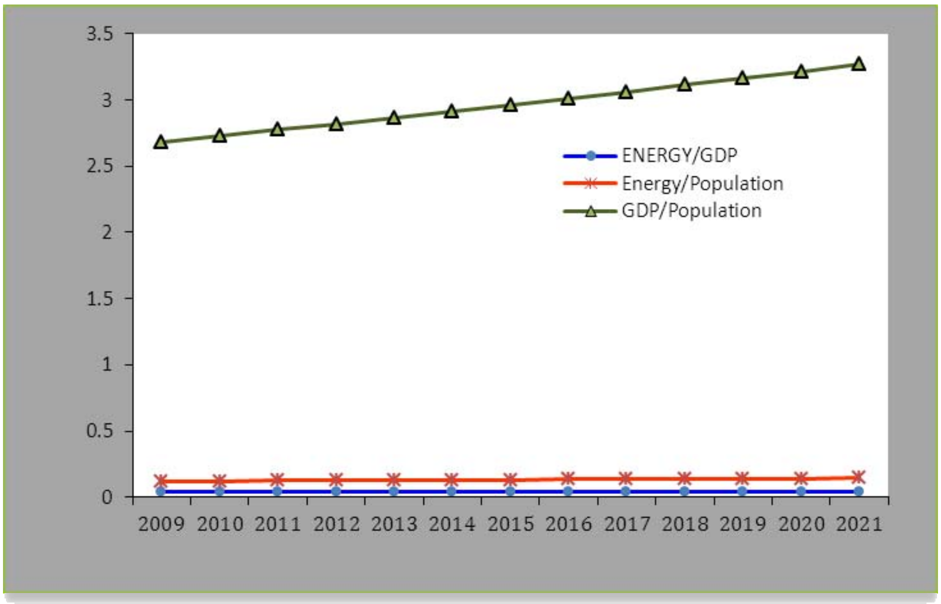

Figure 4: The correlative comparison values of Energy/GDP, Energy/Population and GDP/Population

5 Conclusions

In this paper, we proposed an improved Grey forecasting model which integrates residual modification with Markov chain. The data of GDP, Population and Energy consumption of Iran from 1992 to 2006 were used as forecasted examples. We showed that our model increase the accuracy and also predicted the value of GDP, Population and Energy consumption from 2009 to 2021. Finally, we discussed the development relationship between them in future.

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