

Input-output Table and Input-output Model of Import and Export Internalization

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Abstract

This paper reviews the traditional input-output models and analyzes their potential problems and preliminary improvement research. According to the principle of the national economic accounting system, foreign departments are added in the first quadrant of the traditional input-output table. The biochemical input-output model of import and export solves the import and export problems that have plagued the input-output research, and integrates and expands the advantages of import tabulation matrix, MRIO and other tables and models. It has laid a solid foundation for correctly using input-output data to analyze national economy, regional economy and global economy.

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Keywords: Import, Export, Input-output Table, Input-output Model, Foreign Sector.

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1. Introduction

The input-output method has been fruitful in the decades since its inception, both in theory and in practice. 1968 saw the beginning of the inclusion of input-output accounting as an important component of the System of National Accounts (SNA), and many countries began to produce input-output tables on a regular basis. Input-output tables provide the framework function of national economic accounting. However, imports and exports have plagued the study of input-output tables and models. Studies on imports have tried to solve the problem of imports by placing a separate column for imports in the second quadrant (i.e., traditional input-output tables and models), by placing a matrix of imports in the first and second quadrants, and by placing a matrix of imports in the third quadrant, etc. Studies on exports have divided exports into end-use and intermediate use inter-regional input-output tables and models. These studies have undoubtedly enriched the input-output methods and have certain application value and guidance. However, these studies not only make the form of traditional input-output tables and models complicated and no longer concise, but also some of them make the relevant economic meanings ambiguous, making the studies for the sake of studies and turning them into a numbers game. Based on the principles of the System of National Accounts (SNA), this paper introduces a virtual foreign sector and internalizes both imports and exports to propose an input-output table and input-output model. The table and model have the advantages of traditional input-output tables and models, namely, concise tables and models, symmetrical tables, simple and accurate data sources, and equal data, and at the same time, they solve the import and export problems that have been plaguing input-output research in terms of content, integrate and expand the advantages of tables and models such as import column into matrix and MRIO. It lays a solid foundation for the proper use of input-output data to analyze national, regional and global economies.

2. Traditional input-output tables and models

In the 1930s, based on the previous research on the interdependence of economic activities, Vassily Leontiev, an American economist and professor at Harvard University, compiled the input-output tables of the United States in 1919 and 1929 using the national census data of the U.S. Bureau of the Census. The world's first paper on input-output techniques was published in the *Review of Economics and Statistics* in 1936, i.e., "Input-Output Quantity Relations in the U.S. Economic System", marking the birth of input-output techniques.

Table 1 shows the three-sector input-output table for China in 2000. This traditional input-output table lays the foundation of the input-output research base. For example, the table has three quadrants, the first quadrant is intermediate input/intermediate use, the second quadrant is end use, and the third quadrant is initial input. From the row direction, it represents the use direction of the product, which is divided into two categories, one is intermediate use and the other is end use; from the column direction, it represents the input or consumption of the product,

which is also divided into two categories, one is intermediate input/intermediate consumption and the other is initial input. The traditional input-output table presents aesthetics and makes economic sense. This beauty is reflected in the symmetrical beauty of the external shape of the table and the equivalence of the data within the table. For example, the number of rows and columns in the first quadrant are equal and the data are presented in a square array; the number of rows in the second quadrant and the number of columns in the third quadrant are equal; the sum of the rows equals the sum of the corresponding columns, i.e., the total output of each product equals the total input; the sum of the data in the second quadrant equals the sum of the data in the third quadrant, both of which are equal to the gross domestic product.

Table 1: 2000 input-output table of three sectors in China

		Intermediate use				Final use						Import	Total output
		Primary industry	Secondary industry	Tertiary industry	Total	Resident consumption	Government consumption	Total fixed capital formation	Increase in inventories	Export	Total		
Intermediate input	Primary industry	4036	8799	1149	13984	10956	0	724	385	585	12650	-543	26448
	Secondary industry	5473	97931	16508	119912	21186	0	30946	-501	19268	70899	-18137	172970
	Tertiary industry	1644	17786	11880	31310	12230	11705	954	-8	3346	28227	-1002	58135
	Total	11153	124516	29537	165206	44372	11705	32624	-124	23199	111776	-19682	257553
Initial input	Depreciation of fixed assets	597	8598	5411	14606								
	Workers' compensation	13443	20863	15614	49920								
	Net production tax	415	8889	4108	13412								
	Operating Surplus	841	10103	3465	14409								
	Value added	15296	48453	28598	92347								
Total input		26449	172969	58135	257553								

The traditional input-output table corresponds to two input-output models, a row model that applies direct consumption coefficients and a column model based on direct allocation coefficients. Let us take the row model only as an example.

$$X=(I-A)^{-1} Y \tag{1}$$

Equation (1) is the core model of input-output analysis, ie, the row model, or the demand-pull model, or the Leontief model. Where X is the n*1 order total output column vector; Y is the n*1 order end-use column vector, including not only residential consumption, government consumption, fixed asset investment, and exports, but also imports as a deduction, which represents various demands from

households, enterprises, and the government; I is the unit matrix, i.e., a square matrix with diagonal elements of 1 and other elements of 0; A is the $n \times n$ order direct consumption coefficient matrix. In the process of input-output modeling, the parameterization of direct consumption coefficients is crucial, and the relationship between total inputs and total outputs is critical.

3. Potential problems of traditional input-output tables and models and preliminary improvements

The conventional input-output models and frameworks saw widespread application from their inception. Leonid Hurwicz's notable contribution to this field earned him the Nobel Prize in Economics. In 1968, the System of National Accounts (SNA) incorporated input-output accounting as a crucial component. Subsequently, many nations began to routinely compile input-output tables.

However, in the application of traditional input-output models and frameworks, inherent issues came to light.

Firstly, there is a classification challenge. Within the system of national economic accounts, input-output tables differ from flow of funds tables, balance sheets, and the like. The primary distinction lies in the organizational classification present in the other tables, whereas the input-output table predominantly employs industrial classification. This discrepancy necessitates the construction of social accounting matrices when employing models for general equilibrium analysis based on the input-output table. Moreover, aside from product classification by industry in the first quadrant, the second quadrant classifies units by organizational sectors, such as government departments and household sectors.

Secondly, there is the matter of imports and exports. Traditional input-output models face challenges concerning international trade, a topic this paper will delve into later.

Thirdly, there's the issue of flow. The data of traditional input-output tables and the variables of models are all flow data or flow variables, while the stock data or stock variables are also very important for the growth of national economy. Based on the traditional input-output table, Professor Chen Xikang added land and other natural resources, fixed assets, working fund, and labor as occupancy components to construct an input-occupancy-output table for grain production forecasting, and achieved very significant results in grain production forecasting (Chen, 1992; Chen and Guo, 1996), and some extension studies were subsequently conducted (Chen and Yang, 2003). After more than thirty years of development, the input-occupancy-output technique has evolved from its initial application to crop yield forecasting research to its current application to many fields such as finance, water resources, energy, employment, human capital, education, and environment, and many research results have been achieved.

Lastly, the traditional input-output tables and models are static, which can only reflect the economic development and structure at a certain point in time. The social economy is a constantly changing and developing movement process. So a natural

trend in input-output analysis is to move from a static model to a dynamic model (Zhao et al., 1988; Liew, 2000). The mainstream of the historical process of dynamic input-output technology research can be broadly divided into four stages: the differential form or continuous time model stage, the differential form or discrete time model stage, the stage of considering investment time lags in the model and the stage of dynamic input-occupied output analysis (Liu, 1995).

Regarding the four main issues mentioned earlier, the flow issue and static issue are not inherent problems of traditional input-output tables and models but challenges that arise in applied research when enriching and integrating other factors for a comprehensive analysis of socio-economic issues. Thus, the initial improvements addressing these issues involve augmenting traditional input-output tables and models by adding variables or data. Table 2 provides a typical input-occupancy-output table, showcasing the traditional input-output table in the upper half.

Table 2: General input occupancy output table

		Intermediate use and intermediate occupancy	Final use and final occupancy					Total output and total occupancy	
			1 2 ... n	Consumption	Fixed capital formation	Increase in inventories	Export		Import
					1 2 ... n	1 2 ... n			
Flow input component	Intermediate input	1 2 ... n	X_{ij}	Y_{ik}					
	Final input	Depreciation of fixed assets Workers' compensation Net production tax Operating Surplus	V_{ij}						
	Total input		Q_j						
Inventory occupancy component	Fixed Assets	1 2 ... n	O_{ij}	Y_k^e					O_i
	Inventory	1 2 ... n							
	Financial assets	Currency Deposits Securities Stocks Other							
	Labor	No schooling Primary School Secondary School University degree or above							
	Natural resources	Land Water resource Mineral Forest Other							
	Other	Trademark Patent Other							

But the classification problem and the import and export problem are indeed inherent in traditional input-output tables and models. If we further classify the products in the first quadrant into the non-financial corporate sector and the financial institutions sector, the second quadrant includes the household sector, the government sector and the corporate sector, but only the foreign sector is missing³. The classification problem has not yet attracted much attention, but the import and export problem of traditional input-output tables and models has attracted much attention and given some improvement solutions.

4. The import and export problems of input-output tables and models

In a sense, input-output accounting can be understood as multi-sectoral GDP accounting (Gao, 2018). The D of gross domestic product (GDP) determines the concept of territory, and thus there are bound to be imports and exports as long as it is not a closed economy. However, in the traditional input-output table, the data in the first and second quadrants include both domestically produced goods and foreign produced goods; in the third quadrant, only the initial inputs of the domestic factors are included. As reflected in the input-output model, X (total input and total output) is domestic, but the direct consumption coefficient and final demand include products or demand from abroad. Therefore, in a sense, the traditional input-output tables and models are tables or models that under a segregated economy.

4.1 The Import Problem of Input-Output Tables and Models

Specifically, for the traditional input-output table, imports are placed as a column in the second quadrant, and import data are recorded as negative values. The advantage of the traditional input-output table is more "beautiful", showing that the table is symmetric and the numbers are equal. By table symmetry, we mean that the number of rows and columns are equal in form, and if the number of rows of initial inputs and the number of columns of final use are equal, the input-output table shows diagonal symmetry; by number equality, we mean that the content is equal to total inputs and total outputs, and the sum of the second quadrant and the third quadrant are equal to GDP. This also has obvious disadvantages, namely, although imports are placed in the second quadrant, other data in the first and second quadrants may still include imports; the quadrant where the data in the import's column are located is end-use, but in reality, some are intermediate use.

Some researchers have been critical of input-output models that are based on traditional input-output tables. For example, Dietzenbacher et al. (2005) demonstrated that ignoring the distinction between inputs from different sources causes an overestimation of the sectoral multiplier effect by building an input-output model based on an input-output table with imports listed as a matrix and a traditional

³ The input-output table also lacks the sector of non-profit institutions serving the household sector, but other SAN tables also lack this sector.

input-output model. According to this study, imports should be separated from the original data in the first and second quadrants to form n rows of imports for intermediate use, end use, and exports. This results in the input-output table of the import column into matrix as shown in Table 3.

Table 3: Input-output table of import column into matrix

			Intermediate use	Final use			Total output
			n sectors	Consumption	Investment	Export	
Intermediate input	Domestic products	n sectors	Intermediate consumption of domestic products by the domestic sectors n×n	Consumption of domestic products	Fixed assets and inventories formed from domestic products	Export of domestic products	Total domestic output
	Imported products	n sectors	Intermediate consumption of imported products by the domestic sectors n×n	Consumption of imported products	Fixed assets and inventories formed from imported products	Export of imported products	Total import
Initial input	Depreciation of fixed assets Workers' compensation Net production tax Operating Surplus						
	Total input						

The input-output model with imports listed as a matrix is as follows.

$$X=(I-A^d)^{-1} Y^d \tag{2}$$

$$M= A^m (I-A^d)^{-1} Y^d + Y^m \tag{3}$$

In Eq. (2), X is the n*1 order total output column vector; Yd is the n*1 order end-use column vector, including only residential consumption, government consumption, fixed asset investment, and exports, excluding imports as a deduction, which represents the various demands for domestically produced products from households, firms, and the government; I is the unit matrix, i.e., a square matrix with diagonal elements of 1 and other elements of 0; Ad is the n*n order direct consumption coefficient matrix of domestic products.

In Eq. (3), M is the total import column vector of order n*1, Ym is the import portion of the end-use column vector of order n*1, and Am is the direct consumption

coefficient matrix of imported products of order $n \times n$.

The input-output table and input-output model of import column into matrix overcomes the shortcomings of the traditional input-output table regarding imports and has abundant information, and GTAP and OECD use this form. But there are many problems with this improvement. For example, the difficulty and workload of compiling the table are large, the symmetry of the table is not good enough (the second quadrant has $2n$ rows while the third quadrant has only n columns), the input-output model is not concise enough (two equations are composed and there is no logical relationship between the two equations), and the biggest problem is that the sum of the second quadrant (the sum including imports and the sum excluding imports) is not equal to the sum of the third quadrant, which destroys the balance of the original input-output table. The following demonstrates the existence of this problem.

According to Table (3), the sum of total inputs is equal to the sum of initial inputs in the third quadrant plus the sum of intermediate inputs of imported products plus the sum of intermediate inputs of products in the region, and the sum of total outputs is equal to the sum of end uses of products in the region plus the sum of intermediate uses of products in the region in the second quadrant, while the sum of intermediate inputs of products in the region is equal to the sum of intermediate uses of products in the region, according to the requirement that total inputs are equal to total outputs requirement, then the sum of initial inputs in the third quadrant plus the sum of intermediate inputs of imported products is equal to the sum of end uses of the region's products in the second quadrant, so the sum of end uses of the region's products in the second quadrant is greater than the sum of initial inputs in the third quadrant, or the sum of the second quadrant excluding imports is greater than the sum of initial inputs in the third quadrant, so the sum of the second quadrant (including the sum of imports and or the sum excluding imports) are not equal to the sum in the third quadrant.

To make the sum of the second quadrant equal to the sum of the third quadrant, Sai Liang et al. (2016) put the intermediate inputs of imports into the third quadrant, i.e., the initial inputs include not only the depreciation of fixed assets, labor compensation, net production taxes, and operating surplus of the traditional input-output table, but also the intermediate use of imports. This refinement model simplifies the input-output table and input-output model with imports listed in a matrix, which makes the improved table formally the same as the traditional input-output table on the one hand and presents the characteristics of equal number of table symmetries, on the other hand, the improved input-output model has only one equation, equation (2), which is also formally the same as the traditional input-output model, while the content also overcomes the traditional input-output model sector The disadvantage of overestimation of multiplier effect. However, this model also has two major drawbacks: one is the fragmentation of imports, as the input-output table and input-output model have only intermediate use imports but not final use imports; the other is the lack of clarity of the meaning of the second quadrant, which no longer represents the value added of each industrial sector.

4.2 The export problem of input-output table and model

Compared with the import problem of traditional input-output tables and models, the export problem is not popular. The discussion focuses on the fact that some of the products exported are also used for intermediate use, and thus it is problematic to place all exports in the second quadrant. Therefore, studying where the exported products are used has become a hot topic in the export side of input-output analysis. This is the inter-regional input-output table and its model. The current mainstream model of interregional input-output tables is the Multiregional Input-Output (MRIO) model, which makes homogeneous assumptions on interregional trade coefficients (i.e., any sector supplies the same proportion of each sector to a city) and requires less data information (Wang et al., 2015; Chen et al., 2016). Internationally focused on the global scale between countries, several organizations have developed global scale interregional input-output tables, such as the Eora database developed by Lenzen et al. (2013), the EXIOBASE developed by Tukker et al. (2013), and the EU-funded WIOD database (Timmer et al., 2015). The difficulty and focus of MRIO tables is the estimation of regional trade coefficients, which can be obtained by survey and non-survey methods. The survey method is not suitable for inter-city use because it requires a lot of human and material resources to obtain the inter-regional trade flows of products or services, which is difficult to achieve. The non-survey method can be used to indirectly estimate regional trade coefficients by using suitable models and more accessible data in the absence of a large amount of flow data, and is widely used. Gravity model (GM), which originates from the law of gravity that the gravitational force between two objects is proportional to their mass and inversely proportional to their distance, is one of the most commonly used empirical models to analyze inter-regional trade (Chaney, 2018; Johnson and Noguera, 2017). Some studies use GM directly to estimate inter-regional sectoral economic flows based on bilateral trade surveys to develop inter-regional input-output tables (Chen et al., 2019). Location quotients (LQ) models are commonly used in regional economics, especially in deriving initial regional input coefficients for MRIO (Flegg and Tohmo, 2016). LQ can also be used directly to estimate the degree of industrial agglomeration (Dong et al., 2020).

Table 4 is a schematic table of the multi-regional input-output model. Table 4 divides one industrial sector of the traditional input-output table into several regional industrial sectors, so that the destination of the products of an industrial sector can be divided into intermediate use for the products of the region, intermediate use in other regions, final use in the region and final use in other regions, which solves the problem of final use and intermediate use of export products between regions and has the advantage of trace the product flow relationship between regions. However, it can also be seen from Table 4 that the overall structure is still a traditional input-output table or an input-output table with imports listed in a matrix, and thus the import problem, export problem, table symmetry problem, and number equality problem still exist. These problems are described in detail above and will not be introduced here. Since this model focuses

on the refinement of the traditional input-output table and the export of the model, the following section focuses on the fact that the export problem still exists.

First, as can be seen in Table 4, exports are placed in the second quadrant as a column, and the exports in this column can be used for intermediate uses, so the problem of exports in the traditional input-output table and model still exists.

Second, if some of the regions with the largest share of exports are included in "other regions", the export data are smaller and the export problem seems to be solved because the regions under study are included in "other regions". However, exports in the second quadrant (including end-use goods exported in other regions) are not always a component of GDP, and thus the economic implications of the second quadrant are problematic.

Incidentally, it makes little sense to produce such MRIO tables. A crucial matter in the process of building input-output models based on input-output tables is the parameterization of direct consumption coefficients (Gao et al., 2018). For a single-region input-output table, since the direct consumption coefficients have micro-technological implications, and technology is relatively stable, the direct consumption coefficients determined by technological characteristics are also stable over time and can be understood as constants, and thus the parameterization of the direct consumption coefficients is logical. However, for MRIO table, if there are differences between regions, the direct consumption coefficients between regions will not only depend on the technology of two regions, but also on the technological differences between two regions, and more importantly, the size of trade barriers between regions will affect the direct consumption coefficients between regions, therefore, the direct consumption coefficients of MRIO cannot be parameterized and the input-output model cannot be built; if there are no differences between regions or the differences are small, then these regions can become a whole, and the input-output model can be built with the overall input-output table for comprehensive analysis. In fact, Hebei province is the only province in China where individual counties compile input-output tables, but it is difficult to find papers in the existing literature that study the MRIO model and its application in each region of Hebei province. The MRIO between countries is even less meaningful, as tariff barriers, non-tariff barriers, barriers to factor mobility, differences in policy regimes, cultural differences, etc. make it difficult to parameterize the direct consumption coefficient, and let alone its economic significance⁴.

⁴ For example, the direct consumption coefficient obtained by dividing the value of country A's product by country B's total input has a different economic meaning than the direct consumption coefficient obtained by dividing the value of country A's product by country A's total input, with the former representing country B's technology and the latter representing country A's technology. So does the first quadrant of this mix represent country A's technology or country B's technology?

Table 4. Schematic table of MRIO

		City 1			City 2			City s			Final use			Export	Total output
		sector 1	sector 2	sector j	sector 1	sector 2	sector j	sector 1	sector 2	sector j	city 1	city 2	city s		
city 1	sector 1	z_{ij}^{rs}									F_i^{rs}			EX_i^r	X_j^r
	sector 2														
	sector i														
city 2	sector 1														
	sector 2														
	sector i														
city r	sector 1														
	sector 2														
	sector i														
Import		IM_j^s													
Total output		\bar{X}_j^r													

4.3 Summary of input-output table characteristics for import and export improvements

Through the above analysis, we can summarize the existing input-output tables and models for import-export improvements as shown in Table 5.

Table 5: Related input-output tables and models for import and export of import and export

Input-output table	Traditional input-output tables	Solve the problem of imports exports remain unchanged		Solve the export problem import using the column into the matrix
		Input-output tables for imports in a matrix	Input-output table for intermediate use of initial inputs including imports	MRIO Table
Input-output model	$X=(I-A)^{-1} Y$	$X=(I-A^d)^{-1} Y^d$ $M= A^m (I-A^d)^{-1} Y^d + Y^m$	$X=(I-A^d)^{-1} Y^d$	$x^r = A^{rr} x^r + y^{rr}$ $+ \sum_{s \neq r} A^{rs} x^s + \sum_{s \neq r} y^{rs}$
The number of rows in the second quadrant and the number of columns in the third quadrant	Same	Not the same	Same	Not the same
The sum of the second quadrant and the sum of the third quadrant	Equal, the economic meaning is gross domestic product	Not equal, no economic meaning in the second quadrant	Equal, no economic meaning in both the second and third quadrants	Not equal, no economic meaning in the second quadrant
Sectoral multiplier effect	Overestimation	No overestimation	No overestimation	No overestimation
Import data in the input-output table	Unclear classification, unclear meaning	Classification is clear, meaning is clear	Classification is clear, meaning is clear, but data is incomplete	Classification is clear, meaning is clear
Import variables in the input-output model	Not reflected	Embodied, but not in the core equation	Not reflected	Embodied, but not in the core equation
Export data in the input-output table	Unclear classification, unclear meaning	Unclear classification, unclear meaning	Unclear classification, unclear meaning	Certain classification, certain meaning
Export variables in the input-output model	Embodied, but not clear	Embodied, but not clear	Embodied, but not clear	Embodied, with some clarity
Economic meaning of direct consumption coefficient	Explicit, parameterizable	Explicit, parameterizable	Explicit, parameterizable	$A^{rs}(r \neq s)$ is not clear, parameterization requires conditions

From Table 5, we can see that the current input-output tables and models have some shortcomings in dealing with both imports and exports, and these shortcomings exist essentially because the study of the economic significance of exports and imports is not deep enough.

5. The biochemical input-output table and input-output model of import and export

As can be seen through the analysis in Part III, the endogenous nature of some imports is recognized. This is because this part of imports is used on intermediate inputs. However, in the corresponding input-output model, the internal biochemistry of imports is not fully reflected, and there is no parameter of imports in the key equation (2). This indicates that the internalization of imports is not complete and needs to be further internalized. The direct consumption coefficient calculated from the intermediate inputs in imports still reflects the regional micro-technology quotas, especially under non-competitive conditions where imports are not replaceable, and thus the rate of imported intermediate inputs in a given industrial sector reflects the technological level and production capacity of that industrial sector, and this direct consumption coefficient should enter into the core equation (2).

The analysis in the third part also shows that the endogeneity of some exports is recognized. This is because this part of exports is used on intermediate inputs from outside the region. However, in the corresponding input-output model, the endogeneity of exports is not fully reflected, and there is no parameter for exports in the key equation (2). This indicates that the internalization of exports is not complete and needs to be further internalized.

5.1 Virtual establishment of foreign institutional sector

If a country or region has only exports and imports, then the input-output model cannot be built according to the above input-output table. However, a country or region with only imports and exports can also build an input-output model because the country's imports and exports do not go directly to domestic end-use (imports) and foreign end-use (exports) without any intermediary, but the activity of this intermediary is still a production activity, only that this production activity is not in the country. The end-use product abroad, i.e., export, is an intermediate use for this intermediary, not an end-use.

According to this idea, we add a virtual structural sector or industrial sector to the traditional input-output table, i.e., the foreign institutional sector. This institutional sector is the virtual institutional or industrial sector; the intermediate inputs of this sector are the exports of the corresponding industrial sector; the initial inputs of this sector are the depreciation of fixed assets, labor compensation, net production taxes and operating surplus in the region, etc.; the outputs of this sector are the imports of the region. The value volume of regional imports is essentially determined by the value volume of regional exports. Without exports there is no corresponding foreign

exchange to purchase imports, and without a certain amount of exports certain levels of imports cannot be achieved. Thus, the inputs of the foreign sector are exports and the outputs are imports.

The value of the foreign institutional sector is that it provides the domestic institutional units with imports for end use. Thus, from the row direction, the foreign sector provides imports that provide intermediate use for other industrial sectors and end use for domestic institutional units, respectively; from the column direction, the production of the foreign institutional sector needs to be put into exports for domestic industrial sectors.

The foreign institutional sector still needs to satisfy the assumptions of the Leontief production function of the traditional input-output table and the assumption of homogeneity. Under these two assumptions, the direct consumption coefficients and direct distribution coefficients of the foreign institutional sector can be parameterized and the new input-output model can be built.

5.2 Input-output tables of import and export internalization

Before building the new input-output model, we construct the input-output table of import and export internalization based on the establishment of the virtual foreign establishment sector in 4.1 (as in Table 6).

Table 6: Input-output table of import and export internalization

			Intermediate use		Final use		Total output
			Domestic products	Foreign sector products	Consumption	Investment	
Intermediate input/consumption	Domestic products	n sectors	Intermediate use/consumption of domestic products by the domestic products	Intermediate use (export) of domestic products in foreign sector products / Intermediate consumption of domestic products by foreign sector products	Consumption of domestic products	Fixed assets and inventories formed from domestic products	
	Foreign sector products	n sectors	Intermediate consumption of foreign sector products by the domestic products/Intermediate use of foreign sector products in the domestic products	Intermediate consumption/use of foreign sector products for foreign sector products	Consumption of imported products	Fixed assets and inventories formed from imported products	
Initial input	Depreciation of fixed assets			0			
	Workers' compensation			0			
	Net production tax			0			
	Operating Surplus			Import-export for each sector			
Total input							

As can be seen from Table 6, the foreign sector's initial input is only the operating surplus and its value is equal to the difference between imports and exports. Because the foreign sector is virtual, depreciation of fixed assets, labor compensation, and net production taxes are all zero. However, the inputs to the sector are exports and the outputs are imports, and thus the operating surplus is the difference between imports minus exports.

Comparing Table 6 with the traditional input-output table, we can find that the overall structure is relatively similar, with table symmetry and equal numbers, as explained below.

First, the number of sectors of total output is $2n$ (n sectors in the region + n sectors in the foreign sector), and the number of sectors of total input is also $2n$. The number of total inputs and the number of total outputs are equal, and the number of rows in the second quadrant and the number of columns in the third quadrant are equal. The table has symmetry.

Second, according to the input-output table listed as a matrix and the construction mechanism of the model, the total input of each of the n sectors in the region is equal to its total output; the total output of each of the n sectors in the foreign sector is equal to the imports of the sector, and the total input of each of the n sectors in the foreign sector is equal to the exports of the sector plus the operating surplus of the initial input (imports - exports of the industrial sector), resulting in the total input of each of the n sectors of the foreign sector is also equal to the imports of that sector, and thus the total output of each of the n sectors of the foreign sector is equal to its total input. So, the total input of each sector is equal to its total output, satisfying the first number equality.

Then, based on the fact that the total input is equal to the total output and the number of total inputs and the number of total outputs are equal, we can deduce that the sum of the second quadrant is equal to the sum of the third quadrant satisfying the second equality.

However, the input-output table of import and export internalization overcomes the shortcomings of the traditional input-output table. Specifically, it is explained as follows.

First, since the input-output table of import and export internalization is constructed on the basis of the input-output table with imports listed in a matrix, it overcomes the shortcomings of the traditional input-output table with unclear classification and meaning of import data. For example, the import data are divided into two categories in the input-output table of import-export endogenous biochemistry, one is intermediate use, which is placed in the first quadrant, and the other is end-use, which is placed in the second quadrant, and the classification is very clear; the meaning of import data of intermediate use is very clear in both row direction and column direction, and in row direction, its meaning is intermediate use of foreign sector products in local products, and in column direction, its meaning is intermediate consumption of foreign products in local products.

Secondly, it has been debated whether export data are used for intermediate use or end use. The introduction of foreign sector in the input-output table of import-export

endogenous biochemistry, thus limiting exports to intermediate use, solves the shortcomings of unclear classification and meaning of export data in the input-output table. For example, in the input-output table of import and export internalization, exports are used as intermediate use and placed in the first quadrant with clear classification; the meaning of export data of intermediate use is very clear in both row direction and column direction, in row direction, intermediate use (export) of the region's products in the foreign sector products, and in column direction, intermediate consumption of the region's products by the foreign sector products.

The input-output table of import-export endogeneity not only retains the characteristics of table symmetry and number equality of the traditional input-output table and overcomes the shortcomings of the traditional input-output table, but also further develops the traditional input-output table. This is mainly reflected in the economic implications of the second quadrant and the third quadrant. In the traditional input-output table, the second quadrant and the third quadrant respectively explicate GDP in two dimensions of initial input and end use. The input-output table of import and export internalization further enhances this meaning. In this table, the second and third quadrants respectively explain from the perspective of initial inputs and end-use not only the domestic production of the product (GDP) but also focus on the domestic end-use of the product and the difference between domestic production and domestic end-use. The sum of the second quadrant is equal to the sum of the third quadrant, and its economic meaning is the domestic end use of the product, which is obtained from the meaning of the second quadrant. The sum of the initial domestic inputs in the third quadrant is the gross domestic product of the product, ie, GDP, which is obtained from the principle of spatial qualification of production accounting. The difference between domestic end use and domestic production of the product is reflected in the operating surplus of the foreign sector. If the foreign sector's operating surplus is positive, which means that imports are greater than exports, then the domestic end use of the product is more than the domestic production of the product; if the foreign sector's operating surplus is negative, which means that imports are less than exports, then the domestic end use of the product is less than the domestic production of the product. The converse is also true.

5.3 Input-output model of import and export internalization

After constructing the input-output table with endogenous import and export, we can construct the following input-output model.

$$X=(I-A)^{-1} Y \quad (4)$$

From equation (4), we can see that this model is formally identical to the traditional input-output model equation (1), which achieves a refined simplification of the model. But equation (4) expresses much richer meaning than equation (1). We explain it step by step below.

First of all, there are no interregional economic transactions and both imports and exports are zero in a closed economy. Then equation (4) is not only formally identical to equation (1), but also expresses exactly the same meaning. Even in an open economy, where there are interregional economic transactions and neither imports nor exports are zero, equation (4) is not only formally identical to equation (1) but also expresses exactly the same meaning, if we disregard the sources of inputs and end-uses, while taking exports as an end-use. Therefore, the traditional input-output model is a special case of the input-output model of import and export internalization.

Secondly, we consider different sources of inputs, but still treat exports as an end use. Because the direct consumption coefficient matrix A's the nth + 1 row are all 0, equation (4) decomposes the following two equations.

$$X=(I-A^d)^{-1} Y^d \tag{5}$$

$$M= A^m (I-A^d)^{-1} Y^d + Y^m \tag{6}$$

Where $A=(A^d,A^m)^T$, A^d is the matrix of direct consumption coefficients of the region's products to the region's products, A^m is the matrix of direct consumption coefficients of the region's products to the foreign sector's products, and the meaning of the other variables is the same as in equation (2) (3).

Therefore, in this sense, the input-output model in which imports are listed as matrices is also a special case of the input-output model of import and export internalization.

Next, we assume that there are only two regions of the economy, and assume that region r is the region and region s is the other region. Then, region s is the foreign sector from the perspective of region r, but there is still a difference between region s and the foreign sector. This difference is reflected in the fact that the domestic sector is virtual and thus has no end use, but region s is real and it has end use. Therefore, the input-output model of import and export internalization needs to be adjusted by adjusting the portion of exports from region r to region s for end use to the second quadrant, i.e., the original end use Y^r becomes (Y^{rr}, Y^{rs}) . The direct consumption coefficient matrix A can be decomposed after adjustment as

$$\begin{bmatrix} A^r_r & A^r_s \\ A^s_r & A^s_s \end{bmatrix}$$

Where A^{rr} is the matrix of direct consumption coefficients of products of this region (region r) for products of this region, A^{ss} is the matrix of direct consumption coefficients of products of foreign regions (region s) for products of foreign regions, A^{rs} is the matrix of direct consumption coefficients of products of foreign regions (region s) for products of this region, and A^{sr} is the matrix of direct consumption

coefficients of products of this region (region r) for products of foreign regions. Equation (4) can be decomposed into an equation as follows

$$X^r = A^{rr}X^r + A^{rs}X^s + Y^{rr} + Y^{rr} \quad (7)$$

Comparing equation (7) with MRIO's model, it can be seen that they are exactly the same. But the input-output model using the internalization of exports and imports avoids various difficulties encountered in MRIO's model, such as tariff barriers, the economic implications of the first quadrant, etc.

Therefore, in this sense, the MRIO input-output model can also be evolved from the import-export endogenous input-output model.

To sum up, the input-output model of import-export endogeneity is a general model that can be evolved into other models under certain conditions. Thus, the economic implications of other models can be reflected in the import-export endogenous input-output model, and the import-export endogenous input-output model also has economic implications that cannot be demonstrated by other models.

5.4 Simplified input-output tables and models of import and export internalization

Although imports listed as a matrix can bring a lot of information, it is not necessary to make a matrix of imports according to the process of obtaining import data, but just make a row.

According to the announcement of the General Administration of Customs on the statistical survey on the use of imported goods to go⁵, we can get the total amount and value of imports for intermediate use in a certain industrial sector is determined and well documented, but the specific intermediate consumption in a certain industry is to be estimated by the relevant personnel of the enterprise. Therefore, we take the total amount as the basis from the point of data source. In addition, according to the technical assumptions of the input-output table, we also do not need to break down the imports to the intermediate consumption of a specific industrial sector.

For exports, it is even more impossible to directly obtain the amount of intermediate use in a particular industrial sector abroad according to the available information, and the results calculated using the model are negotiable. But the total exports of a particular domestic industrial sector are determined and well documented.

In view of the above two points, we construct a simplified input-output table of import and export internalization as shown in Table 7. There is only one foreign sector in Table 7, and the number of total outputs is $n+1$ and the number of total inputs is $n+1$. All the data in Table 7 can be obtained from the statistics of the region at this time, which guaranteeing the data quality. Compared with other improved input-output tables, this table has less workload and reliable data quality.

⁵ <http://www.customs.gov.cn/customs/302249/302266/302267/3691216/index.html>

In view of the above two points, we construct a simplified input-output table of import and export internalization as shown in Table 7. There is only one foreign sector in Table 7, and the number of total outputs is $n+1$ and the number of total inputs is $n+1$. All the data in Table 7 can be obtained from the statistics of the region at this time, which guaranteeing the data quality. Compared with other improved input-output tables, this table has less workload and reliable data quality.

Table 7: Simplified input-output tables of import and export internalization

			Intermediate use		Final use		Total output
			Domestic products	Foreign sectors	Consumption	Investment	
Intermediate input/consumption	Domestic products	n sectors	Intermediate use/consumption of domestic products by the domestic products	Export of domestic products / Intermediate consumption of domestic products by foreign sector products	Consumption of domestic products	Fixed assets and inventories formed from domestic products	
	Foreign sectors	1 sector	Intermediate consumption of foreign sector products by the domestic products/Intermediate use of foreign sector products in the domestic products	Exports in imports	Consumption of imported products	Fixed assets and inventories formed from imported products	
Initial input	Depreciation of fixed assets			0			
	Workers' compensation			0			
	Net production tax			0			
	Operating Surplus			Total import-total export			
Total input							

Correspondingly, the simplified input-output model of import and export internalization is still Eq. (4), except that the dimensions of X and Y are reduced by nearly half in Eq. (4), from the original $2n$ to $n+1$; A is reduced from $(2n)*(2n)$ to $(n+1)*(n+1)$.

6. Conclusion and application

The simplified input-output table and model of import and export internalization are compared with other tables and models, and the results are shown in Table 8. We can see through Table 8 that the conclusion of this paper is that the input-output table and model of import and export internalization has the advantages of traditional input-output table and model, i.e., concise table and model, symmetrical table, simple and accurate data source and equal data. At the same time, it solves the import and export problems that have been plaguing input-output research, and integrates and extends the advantages of tables and models such as the import column matrix and MRIO.

Based on the research process and related findings in this paper, the application of this table and model can be discussed as follows.

Firstly, the input-output table and model of import-export endogenous are very consistent in form with the traditional input-output table and model, so the problems studied by using the traditional input-output table and model can be studied in this problem with compatibility.

Secondly, the input-output table and model with internalized import and export are internalized through the virtual foreign sector, which broadens the traditional thinking pattern and promotes the in-depth understanding of the economic meaning of import and export to the regional economy, and further explores the value of input-output table.

Thirdly, since the input-output table and model of import and export internalization solve the import and export problem that has been plaguing input-output research in terms of content, it overcomes the problem of overestimation of the sectoral multiplier effect of the traditional model due to the import and export problem.

Finally, the foreign sector is used as a bridge to regional input-output and thus to study inter-regional industrial linkages. A solid foundation is laid for the proper use of input-output data to analyze the national, regional and global economies.

Table 8: Comparison of simplified input-output tables and input-output models of import and export internalization with other tables and models

Input-output table	Traditional input-output table	Simplified input-output tables and input-output models of import and export internalization	Input-output table with imports listed as a matrix	Input-output table for initial inputs including imports	MRIO table
Input-output model	$X=(I-A)^{-1} Y$	$X=(I-A)^{-1} Y$	$X=(I-A^d)^{-1} Y^d$ $M= A^m (I-A^d)^{-1} Y^d + Y^m$	$X=(I-A^d)^{-1} Y^d$	$x^r = A^{rr} x^r + y^{rr}$ $+ \sum_{s \neq r} A^{rs} x^s + \sum_{s \neq r}$
Solving problems	None	Solve import and export problem	Solve the import problem, export remains unchanged		Solve the import and part of the export problem
The number of rows in the second quadrant and the number of columns in the third quadrant	Same	Same	Not the same	Same	Not the same
The sum of the second quadrant and the sum of the third quadrant	Equal, the economic meaning is gross domestic product	Equal, the economic meaning is domestic end use	Not equal, no economic meaning in the second quadrant	Equal, no economic meaning in both the second and third quadrants	Not equal, no economic meaning in the second quadrant
Sectoral multiplier effect	Overestimation	No overestimation	No overestimation	No overestimation	No overestimation
Import data in the input-output table	Unclear classification, unclear meaning	Classification is clear, meaning is clear	Classification is clear, meaning is clear	Classification is clear, meaning is clear, but data is incomplete	Classification is clear, meaning is clear
Import variables in the input-output model	Not reflected	Embodied in the core equation	Embodied, but not in the core equation	Not reflected	Embodied, but not in the core equation
Export data in the input-output table	Unclear classification, unclear meaning	Clear classification, clear meaning	Unclear classification, unclear meaning	Unclear classification, unclear meaning	Certain classification, certain meaning
Export variables in the input-output model	Embodied, but not clear	Embodied in the core equation	Embodied, but not clear	Embodied, but not clear	Embodied, with some clarity
Economic meaning of direct consumption coefficient	Explicit, parameterizable	Explicit, parameterizable	Explicit, parameterizable	Explicit, parameterizable	$A^{rs}(r \neq s)$ is not clear, parameterization requires conditions
Can the available data support	Yes	Yes	Research or model assumptions are required	Research or model assumptions are required	Research or model assumptions are required

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