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Pricing Rent for Social Housing Under Uncertainty

Sheng-Hau Lin[1] and Chia-Tsong Chen[2]*

Abstract

Setting appropriate rent levels for social housing facilitates policy implementation and the effective allocation of resources for social programs. The conflict between pricing rents at the lowest possible level and the principle of self-sufficiency is an indication that market mechanisms must be considered in pricing rent for social housing. Based on the principle of self-sufficiency and uncertainty in the rental market, we adopted the Samuelson–McKean model to explore the optimal rent for social housing under uncertainty. Simulation results of Taipei, Taiwan indicate that districts with higher uncertainty have a higher threshold of rental income. The rent threshold ranged from 9.16% (Daan District) to 178.39%. (Neihu District) when uncertainty was considered in the model. These results indicate the importance of uncertainty in pricing and evaluating social housing programs. Sensitivity analysis verified that our results are consistent with findings in the literature on real options analysis. This implies that the robustness of the model and of the fact that it takes uncertainty into account make it suitable for pricing the rent of social housing.

JEL classification numbers: D81, R31, R32
Keywords: Housing Policy, Pricing Rents, Uncertainty, Taiwan.

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[1] School of Government, Nanjing University, Nanjing City, China.
[2*] Department of Architecture and Urban Design, Chinese Culture University, Taipei City, Taiwan. Corresponding author.

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1. Introduction

Social housing is an important tool for the implementation of policy pertaining to guaranteed housing. Having the government set the rent levels for social housing facilitates policy implementation and assists in the allocation of resources for social housing based on the principles of sustainable development (Xiao et al., 2016). In most European countries, governments are seeking to make social housing programs more self-sufficient.[3] They are also combining social housing with income-related subsidies to support poorer households (Kathleen et al., 2015). In other words, social housing can be considered a long-term program to promote self-sufficiency (Hua et al., 2009). Below-market rents are a key feature of social housing (United Nations, 2006; Kathleen et al., 2015). However, this necessitates subsidies for investment in social housing and affects the maintenance of dwellings and the mobility of households (United Nations, 2006; Gruis and Nieboer, 2007). This means that there is a conflict between pricing rent at the lowest possible level and the principle of self-sufficiency.

An understanding that market demand and social housing are required to make a profit has led to the concept of market orientation in the management of social housing (Gruis and Nieboer, 2007). In many countries, regulations pertaining to the cost of social housing have been introduced in the shadow of market orientation (Tang, 2008; Tang, 2011). Hills (2007) reported that the rent structure for social housing should take into account local and/or regional variability in the housing market. A study of critical factors influencing the rents of public rental housing (PRH) by public private partnerships (PPPs) in Nanjing of China also indicated the market rent is a significant factor (Yuan et al., 2017).

Uncertainty is a critical factor in evaluating and pricing property, and must therefore be taken into account when addressing the issues of social housing. Most investment decisions are characterized by partial or complete irreversibility, uncertain return on investment, and deferrable investment timing (Dixit and Pindyck, 1994). Social housing program is no exception. Real options analysis (ROA), which takes into account the effects of uncertainty, could be used in the evaluation of social housing program. Most ROA studies on the property market have addressed the time value for undeveloped land, including Quigg (1993), Chiang et al. (2005), Grovenstein et al. (2011). The timing of land development was also addressed by Capozza and Helsley (1990), Capozza and Sick (1994), Cunningham (2006), Bulan et al (2009) and Wang et al. (2016). Peng et al. (2010). Chen and Lai (2013) studied the issues of real estate redevelopment. Hui and Lau (2011) addressed the viability of building rehabilitation whereas Hsieh and Lin (2016) discussed pricing for leasing non-public-use land.

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[3] The principle of self-sufficiency stipulates that social housing cover capital and operating costs through the rental income itself; i.e., without external assistance from the government or other organizations.

The government of Taiwan is seeking strategies to promote social housing based on the principle of self-sufficiency and price rents appropriately in a resource-limited environment. Taiwan provides a suitable background for a discussion on rent pricing for social housing. In 1975, the Taiwanese government introduced the Public Housing Act to solve the problem of housing; however, funding and quality issues hindered progress. The government has since sought to stimulate short-term demand in the real estate market through mortgage rate subsidies; however, this has increased housing prices and homelessness. In 2012, under the pressure of presidential elections, the government passed the Housing Act stipulating that social housing be built by the government or private sectors and used for rental purposes (Chang and Yuan, 2013). This has greatly increased the availability of social housing witnessed. A total of 6,813 houses were provided exclusively for rent (not for sale), which accounts for 0.8 % of the 850,000 households without ownership below the 50 % quantile of the entire country. Clearly, Taiwan has considerable demand for social housing. In 2016, President Tsai Ing-wen and her ruling party proposed the construction of 200,000 social housing units in the following 8 years. ROA has been extended to social housing (Ho, et al., 2009; Li et al., 2014; Li et al., 2016); however, relatively few studies have examined this issue from the perspective of the government and researchers have yet to propose a model of the rents of social housing that takes market mechanisms into account. In this study, we developed a comprehensive evaluation and pricing model for social housing based on the principle of self-sufficiency and market mechanisms (i.e., considering uncertainty in the housing market) from the perspective of the government. We adopted the Samuelson–McKean model, which has previously been used to evaluate of the timing of real estate development (Ho et al., 2009; Peng et al., 2010; Hui et al., 2011; Chen and Lai, 2013; Yao and Pretorius, 2014; Wang et al., 2016) and asset pricing (Hsieh and Lin, 2016) to conduct simulations comparing the value of investments and rental thresholds in 12 districts of Taipei, Taiwan. This study design is based on the concept proposed by Peng et al. (2010). We used sensitivity analysis to explore factors that influence the pricing of rents, investment value, and rental thresholds.

The remainder of this study is structured as follows. Section 2 reviews rent pricing for social housing and the use of ROA to investigate issues related to social housing. Section 3 presents the pricing model for social housing. Section 4 describes the empirical materials and methods. Section 5 presents our results and a comparison of rental thresholds in 12 districts of Taipei as well as sensitivity analysis of the parameters influencing rent pricing for social housing. Conclusions and policy implications are listed in the final section.

2. Literature Review

Kathleen et al. (2015) reported that social housing programs can be made more self-sufficient by combining them with income-related subsidies to support poorer households. This is the policy direction adopted by most of the governments in

Western Europe, where the rents of social housing are typically determined by the total cost of the program (i.e., cost rent principles or cost-based rents). This means that rental incomes help to balance the accounts of non-profit entities, or provide an expected return on investment after paying back the subsidies (United Nations, 2006; Whitehead, 2014b; Cahill, 2014). The rents in England and the Netherlands are based on the value of the dwelling (in terms of consumer demand and/or values in other sectors of the housing market) or on the incomes of tenant households (Whitehead, 2014), while the rents in Ireland are based on individual incomes (United Nations, 2006).

Below-market rent is a key feature of social housing (United Nations, 2006; Kathleen et al., 2015). Pricing rent at the lowest possible level can lessen the need for housing allowances; however, this necessitates heavy capital investment and elevated operating costs. Furthermore, a failure to encourage households to adapt housing consumption to their needs leads to inefficiencies (United Nations, 2006). In other words, the regulation of rents can also have an impact on the maintenance of dwellings (Gruis and Nieboer, 2007). A significant difference between social housing/sector rents and market rents can also have a negative impact on mobility (United Nations, 2006).

Denmark, Germany, and Austria must deal with inefficiencies in investments for social housing, where costs have clearly been inflated as there is nothing to ensure that costs are set at their minimum (Whitehead, 2014). This has led to the introduction of market orientation into the evaluation of social housing. Essentially, this means that social housing must make a profit in order for the government to fulfill social objectives (Gruis and Nieboer, 2007). Rents are related to the quality and position of housing in the market, based on market demand and tenant preferences (Gruis and Nieboer, 2007). Hills (2007) claimed that the rent structure for social housing should reflect local and regional variability in the housing market. Yuan et al. (2017) identified six factors influencing the rents of public rental housing (PRH) by (public private partnerships) PPP, including market rents.

The need to make social housing self-sufficient (i.e., lower the operating expenses and related subsidies) in line with market mechanisms (Hill, 2007; Tang, 2008; Tang, 2011; Yuan et al., 2017) makes ROA suitable metric by which to evaluate social housing programs and the pricing of rents.

Investment decisions pertaining to social housing are characterized by partial or complete irreversibility, uncertain return on investment, and deferrable investment timing (Dixit and Pindyck, 1994). The traditional net present value (NPV) and the discounted cash flow (DCF) lead to a contradiction of investment projects between the evaluation stage and actual investment stage (Fernades et al., 2011). ROA is considered a more appropriate method by which to evaluate the feasibility of an investment (Zhu 2012). Myers (1977) claimed that enterprise value is influenced by the NPV of existing investment projects as well as future options, thereby introducing the financial option to the evaluation of real assets. Bernnan and Schwartz (1985) applied the option to evaluate investments in natural resources. Other studies have focused on its applicability to investments in oil (Smith and Nau, 1995; Smith and McCardle, 1998; Fan and Zhu, 2010).

The evaluation and pricing of real estate can be influenced by uncertainty in the market. The application of ROA in studies related to real estate have addressed a number of issues, including the time value for undeveloped land (Quigg, 1993; Chiang et al., 2005; Grovenstein et al., 2011), the timing of land development (Capozza and Helsley, 1990; Capozza and Sick, 1994; Cunningham, 2006; Bulan et al, 2009; Wang et al., 2016), the option value of real estate redevelopment (Peng et al., 2010), the viability of building rehabilitation (Hui and Lau, 2011), and redevelopment projects (Chen and Lai, 2013). Other researchers have looked at pricing for the leasing of nnon-public-use land from the perspective of governments (Hsieh and Lin, 2016). Studies related to decision-making in land development have used the volatility of house prices as a proxy variable for uncertainty (Quigg, 1993; Chiang et al., 2005; Cunningham, 2006; Bulan et al, 2009; Grovenstein et al., 2011; Wang et al., 2016), whereas the volatility of rents has been used in studies related to the timing of real estate redevelopment (Peng et al., 2010). The pricing of social housing programs oriented toward leasing could also be used as a proxy for the volatility of rents in real estate markets.

ROA has recently been applied to issues associated with social housing, such as the mobility of tenants (Ho et al., 2009) and the evaluation of public rental housing provided by private developers (Li et al., 2014; Li et al., 2016). Ho et al. (2009) used the binomial real option pricing model and the Samuelson–McKean closed-form solution to investigate the effects of government subsidies on option premiums for social housing tenants, while taking into account the main upgrading program (MUP) implemented by the Singapore Housing Development Board (HDB) wherein the owner holds a call option to upgrade flats under MUP policy. This provides evidence that government subsidies had a significantly impact on option premiums of the MUP. Li et al. (2014) and Li et al. (2016) utilized the Black-Scholes model (B-S model) and Cox-Ross-Rubenstein binomial option pricing model (CRR BOPM) to evaluate the feasibility of real options analysis on PRH provided by private developers. Li et al. (2014) compared conventional NPV methods and ENPV (i.e., adding an option premium to the traditional NPV), revealing a negative NPV indicator and a positive ENPV indicator. Furthermore, sensitivity analysis revealed that increasing the average rent of PRH buildings was the most effective means of enhancing investment value under an ENPV situation. Li et al. (2016) also presented the concept of Building Own–Operation–Concession (BOOC) mode into investment evaluation in public rental housing from the perspectives of private developers. Private developers retain rights in the concession contract of BOOC to undertake fund-raising, construction, operations, and demolition at any time within the contract period. The authors indicated that CRR BOPM for BOOC motivates private developers to invest in and manage projects. This evaluation method also reflects the potential investment value of PRH to a considerable extent.

Despite efforts to apply ROA to social housing, most of this work has focused on the influence of subsidies on option premiums for tenants of social housing (Ho et al., 2009), or the influence of options (such as fund-raising, construction, operation, and even demolition) on the option premiums when evaluating investments in PRH by private developers (Li et al., 2014; Li et al., 2016). a number of research gaps remain. Few researchers have addressed the use of ROA to evaluate the feasibility of social housing programs from the perspective of governments. Li et al. (2014) reported that increasing the rents of social housing could promote the development of social housing by private developers; however, researchers have yet to formulate a model by which to determine appropriate rent prices model as a reflection of market forces.

3. Modeling Optimal Rents for Social Housing under Uncertainty

Governments may choose to build or not to build social housing based on ROA. This decision is influenced by six factors (Ho et al., 2009):

1. the price of the underlying property (i.e., the present investment value of social housing), which represents the present value of cash flow expected from the investment opportunity.
2. exercise price, which represents the present value of exercising the option (i.e., investment costs of social housing).
3. uncertainty, which represents unpredictability in future cash flow (i.e., the uncertainty of rent changes of social housing under).
4. time to expiration of option (i.e., time to expiration of option held by the government to build or not to build social housing)
5. dividends paid.
6. risk-free interest rates.

The government may make investments at any time prior to the expiration date of the option, as in the case of a government leasing non-public-use land (Hsieh and Lin, 2016). Thus, we adopted the Samuelson–McKean method in the construction of our model, which accounts for the price of options in the evaluation of investment projects. This approach also suits the optimal timing of developments in the property market (Ho et al., 2009). This model has previously been applied to the evaluation of social housing in Singapore in cases where tenants may or may not choose to be upgraded (Ho et al., 2009). It has also been applied in determining the real option value of urban redevelopment (Peng et al., 2010), the viability of building rehabilitation (Hui et al., 2011), pricing for long-term leases in Hong Kong (Yao and Pretorius, 2014), and pricing for non-public-use land (Hsieh and Lin, 2016).

We assumed that social housing would be taken over and run by the government following the completion of construction, unlike the PRH situation in China (Li et al., 2014; Li et al., 2016).

The current value of investments in social housing is S at t, and the value of investments including the building cost is I. We assumed that the annual rental income of social housing is X, which is the only parameter that changes with time in compliance with Arithmetic Brownian motion. The current underlying investment value of social housing omitting construction costs and uncertainty (i.e., under certainty), is calculated as follows:

$$I = \left[\frac{X}{r-\delta}(1-e^{-(r-\delta)T}) + R \right] \times (1-\gamma) \tag{1}$$

When building and land use costs are considered, the value of underlying investment for social housing is calculated as follows:

$$S = \left[\frac{X}{r-\delta}(1-e^{-(r-\delta)T}) + R - K \right] \times (1-\gamma) \tag{2}$$

where r is the risk-free rate, δ is the growth rate for rent of social housing per unit time, e is the natural logarithm, T is the service life of social housing, R is the residual value of social housing, and K is the total building cost. Hua et al. (2009) reported that the provision of social housing by private developers is another approach to the creation of social housing. The *Act for Promotion of Private Participation in Infrastructure Projects* in Taiwan lists social housing as a class of infrastructure project. Li et al. (2014) reported that local governments in China provide tax incentives for such projects and space for commercial projects with the aim of attracting investment from private developers for social housing programs on the build–operate–transfer (BOT) model. Thus, γ represents government incentives for construction by private developers.

Accordingly, the threshold of rental income of social housing under certainty is calculated as follows:

$$X = \left[\frac{(K-R)(r-\delta)}{(1-e^{-(r-\delta)T})} \right] \times (1-\gamma)^{-1} \tag{3}$$

We assumed that when investment uncertainty is introduced into the model, the option of governmental investment in social housing is elastic. Accordingly, the threshold of rental income for social housing complies with arithmetic Brownian motion, as follows:

$$dX = \delta X dt + \sigma X dz \tag{4}$$

where σ is the instantaneous conditional expected %age change in investment income from social housing per unit time (dz is an increment of a standard Weiner process). To solve the equation, we first define P as the investment value of the underlying social housing under uncertainty. Using Ito's Lemma for the derivation, we derived the following partial differential equation:

$$\frac{1}{2}\sigma^2 X^2 P'' + \delta X P' - rP = 0 \tag{5}$$

This equation was solved by setting the following three boundary conditions (Hsieh and Lin, 2016): 1) $PV(0) = 0$, which is the process of rent threshold absorbing barriers, where the minimum rent threshold is 0; 2) smooth-pasting conditions (indicated by Eq. 6), which refer to the investment value calculated using the optimal rent threshold; 3) high-contact condition, which is $V'(X^*) = \frac{1-e^{-(r-\delta)T}}{r-\delta}$, referring to the maximum investment calculated using first-order differential equation of the optimal rent threshold of social housing:

$$P(X^*) = \left[\frac{X}{r-\delta}(1-e^{-(r-\delta)T}) + R - K\right] \times (1-\gamma) \tag{6}$$

The equations must comply with $V(P) = a_1 P^{\beta_1} + a_2 P^{\beta_2}$, where β_1 >0 and β_2 <0. We assumed that the optimal rent threshold is not less than 0, which means that the equation can be simplified as $V(P) = a_1 P^{\beta_1}$. For the sake of clarity, we replace β_1 with β, which is defined as the elasticity of the option; i.e., the % change in the option value that is not exercised, and associated with a 1% change in the value of an underlying asset (Ho et al., 2009; Hui et al., 2011). Moreover, the %age change in the value of the option must exceed 1 (Hsieh and Lin, 2016).

$$\beta = \frac{-\left(\delta - \frac{1}{2}\sigma^2\right) + \sqrt{(\delta - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}}{\sigma^2} > 1 \tag{7}$$

Thus, we also obtain the value of investment in social housing built by the government under different scenarios, based on the optimal threshold of rental income for social housing, as follows:

$$PV(X(t)) = \begin{cases} a_1 P^{\beta_1} & X < X^* \\ \left[\frac{X}{r-\delta}(1-e^{-(r-\delta)T}) + R - K\right] \times (1-\gamma) & X \geq X^* \end{cases} \tag{8}$$

$$a_1 P^{\beta_1} = \left(\frac{1}{\beta-1}\right)\frac{(K-R)(r-\delta)}{(1-e^{-(r-\delta)T})} \times (1-\gamma)^{-1}\left(\frac{X}{X^*}\right)^{\beta} \tag{9}$$

where $a_1 P^{\beta_1}$ is the value of options for deferrable investments in social housing when the optimal rent threshold is unachieved.

Based on the above equations, we can conclude from a theoretical perspective that the optimal threshold of rental income under uncertainty is X^*, and $\beta/(\beta-1)$ is defined as the hurdle benefit-cost ratio. In other words, we propose that the optimal threshold for social housing under uncertainty is $\beta/(\beta-1)$ times the rent threshold of social housing under certainty.

$$X^* = \left(\frac{\beta}{\beta-1}\right)\frac{(K-R)(r-\delta)}{(1-e^{-(r-\delta)T})}(1-\gamma)^{-1} \tag{10}$$

4. Materials and Methods

To compensate for a lack of empirical data from Taiwan, we employed statistical analysis to verify the effects of uncertainty and building costs on investment value and rent pricing for social housing. Simulations were used to calculate and compare the value of investment and rent thresholds for a variety of scenarios in 12 districts of Taipei. specifically, we sought to estimate the rent thresholds under certainty and uncertainty, and then compare current market rent values, rent values under certainty, and optimal rent values under uncertainty. We also employed sensitivity analysis in exploring the effect of factors influencing rent (including volatility, risk-free rate, building cost, residual value, and incentive of construction for private developers) in cases with investments of various value and rent threshold.

To enable a comparison of rent thresholds and investment values in each district, we referred to Article 28 of the Housing Act of Taiwan. This article stipulates that social housing built by the private sector in urban regions must cover at least 500 square meters of land with at least 600 square meters of total floor area. This standard was adopted in this study. Based on the construction and parcel conditions proposed in non-academic articles by Hua et al. (2009), each district must include 500 ping of land (1,653 square meter: 1 ㎡ = 0.3025 ping) for residential areas established by government or state-owned institutions. With the inclusion of incentives by the government, the floor area ratio can reach 400 %. In this study, we assumed that the maximum floor area is 2,000 ping (6,612 square meter) and the service life of the building is 50 years, in cases where the government owns the rental income from social housing for a period of 50 years. The other parameters of the model are described separately in the following (see Table 1).

4.1 Risk-free rate (r)

Related studies have utilized the government bond yield or exchange fund notes as the risk-free rate. Ho et al. (2009) used the average 5-year Singapore government bond yield and Hui and Lau (2011) used exchange fund notes from the Hong Kong Monetary Authority (HKMA) to evaluate changes in the ownership of social housing and viability of building rehabilitation. The government bond yield in Taiwan is not suitable for use as the risk-free rate due to a lack of complete data over the long-term. Thus, we used the 1-year fixed deposit rate from five major banks in January 2016 as the risk-free rate (Hsieh and Lin, 2016).

4.2 Growth rate for rent (δ) and its volatility (σ)

Due to insufficient data pertaining to the price index at the district scale, we based the growth rate and its volatility for social housing on the method proposed by Peng et al. (2010) to derive the rents of private housing market in each district of Taipei from 2001 to 2013. The data was obtained from the Tsuei Ma Ma Foundation for Housing and Community Services We adopted the method proposed by Hsieh and Lin (2016) for calculating the annual growth rate and fluctuation in rentals based on the consumer price index. As shown in Table 1, Nangang District has the highest annual growth rate (1.738 %) among the districts, followed by Neihu District (0.739 %), whereas Datong District has the lowest annual growth rate (-0.636 %). In terms of volatility, the overall growth rate was between 3 % and 8 %. Nangang District presented the highest growth rate (8.262%) among the districts, followed by Songshan District (5.954%), and Zhongshan District (5.212%). Daan District presented the lowest annual growth rate (3.040%).

4.3 Building and land use cost (k) and residual value (R)

Li et al. (2014) evaluated a social housing project built and run by a private developer in Nanjing, China. That study took into account many of the costs, including the cost of land acquisition, pre-construction engineering expenses, construction and installment expenses, infrastructure, unknown and unpredictable hidden costs, indirect expenses, management costs, and financial costs. The social housing addressed in this study is limited mainly to buildings. Peng et al. (2010) and Hua et al. (2009) proposed dividing costs into building costs, (i.e., including planning and design expenses, construction costs, and engineering expenses), management costs, loan interest, and costs of long-term maintenance and management. We assumed that the building cost is NTD 80,000 per ping, which comes to a total of NTD 160,000,000. The loan interest was calculated by multiplying the total building cost by 1.5%, 2.8% of 1-year fixed deposit rate from five major banks in January 2016, and 2 years is NTD 9,094,400 in total. The cost of long-term maintenance and management per ping (such as operation and housing tax) is NTD 20,000, which comes to a total of NTD 40,000,000. Several studies have reported that lowering the cost of land acquisition is key to promoting social housing. Land planning systems and patterns of ownership have been implemented by the government to promote social housing in Australia and England (Whitehead, 2007; AusTin 2014). Chui (2007) reported that the Hong Kong government retains the ownership of land and development rights in order to subsidize producers for social housing. In this study, land use costs included land rents and taxes based on land value. In accordance with Taipei's public-land leasing laws, land rent is calculated as 5% of the declared land value per ping in 2016[4] multiplied by 2,000 (land areas) over 50 years (land use term). The total land value tax was obtained by multiplying the declared land value per ping in 2016 by 2,000 over 50 years, and 2

[4] As the declared land value from the scale of district is limited, we adjust the announced current land value (it is generally considered to be 80% of the land market price) in 2007 obtained from Peng et al. (2011) to announced land current value in 2016, according to the magnitude of the annual growth reported by Taipei's government (http://land.gov.taipei/ct.asp?xItem=46058399&CtNode=85188&mp=111001). Further, the announced current land value in 2016 is multiplied by 0.375 to obtain the announced land value (announced land value is generally considered to be 30% of the land market price, and hence 0.3/0.8=0.375). Based on this, the declared land value is 80% of the announced land value, where the declared land value is obtained from the announced land value multiplied by 80%.

% of preferential tax rate for owner-occupied residence. The residual value was assumed to be 10 % of the building costs.

4.4 Incentive of Construction for private developers (γ)

Whitehead (2007) and Austin et al. (2008) reported on land planning systems and patterns of ownership aimed at promoting social housing in Australia and England. In this study, we assumed that land was provided by the government free of charge; however, total building costs still present financial challenges that could hinder the development of social housing.

Local governments in China tend to provide tax incentives when implementing social housing based on BOT. They designate a portion of the space for commercial use in order to attract investments in social housing from private developers (Li et al., 2016). In recent years, the Act for Promotion of Private Participation in Infrastructure Projects in Taiwan has included social housing as a type of infrastructure project. Moreover, the Housing Act provides tax exemptions, financing, and volume rewards for private developers involved in the construction of social housing. We adopted the incentives provided for private developers as a variable under the assumption that 60% of the present value of social housing is used to compensate private developers for the costs they incurred in the construction of social housing. The remaining 40 % is allocated to the government (i.e., landholder). This assumption follows the commonly Taiwanese practice of splitting the value of real estate development.

Table 1: Parameter estimation and assumption

Parameter	Definition	Estimated or assumed value			Source
Risk-free Rate (r)	One-year fixed deposit rate from five major banks in January 2016	1.21%			Central Bank of the Republic of China
Growth rate for rent (δ) and its volatility (σ)	Annual growth rate of apartment rent per ping and its volatility in each district in Taipei from 2001 to 2013 (Peng et al. [20] and Hsieh and Lin [23])	**District**	δ	σ	TSUEI MA MA Foundation for Housing and Community Services and the study's calculation based on Hsieh and Lin (2016)
		Zhongzheng District	24.900%	4.602%	
		Datong District	63.600%	4.900%	
		Zhongshan District	61.400%	5.212%	
		Songshan District	30.000%	5.954%	
		Daan District	40.300%	3.040%	
		Wanhua District	4.500%	4.610%	
		Xinyi District	35.300%	3.748%	
		Shilin District	5.500%	4.073%	
		Beitou District	33.100%	3.606%	
		Neihu District	73.900%	3.604%	
		Nangang District	173.800%	8.262%	
		Wenshan District	38.200%	3.542%	
Building and land use cost (k) and residual value (R)	Building costs	NTD 160,000,000			Peng et al. (2011), Hua et al. (2009) and calculation from this study
	Loan interest	NTD 9,094,400			
	Costs of maintenance and management in long term	NTD 40,000,000			
	Land use cost (land rents and land value tax)	**District**	**Land Use Cost (NTD)**		
		Zhongzheng District	3,590,705,908		
		Datong District	2,586,233,852		
		Zhongshan District	2,658,765,724		
		Songshan District	2,568,745,233		
		Daan District	4,059,128,365		
		Wanhua District	1,926,395,028		
		Xinyi District	2,710,429,277		
		Shilin District	631,233,077		
		Beitou District	523,540,382		
		Neihu District	876,077,324		
		Nangang District	759,113,511		
		Wenshan District	709,294,650		
	Residual value	**District**	**Residual value (NTD)**		
		Zhongzheng District	359,070,591		
		Datong District	258,623,385		
		Zhongshan District	265,876,572		
		Songshan District	256,874,523		
		Daan District	405,912,836		
		Wanhua District	192,639,503		
		Xinyi District	271,042,928		
		Shilin District	63,123,308		
		Beitou District	52,354,038		
		Neihu District	87,607,732		
		Nangang District	75,911,351		
		Wenshan District	70,929,465		
Incentive of construction (γ)	Certain %age of investment value	60%			The study's assumption

Note: This table depicts the parameter, its definition, estimated or assumed value and source, in which risk free rate is based on one-year fixed deposit rate from five major banks in January 2016 from Central Bank of the Republic of China. Calculation of growth rate of rent and its volatility is based on Hsieh and Lin (2016), and its data resource from TSUEI MA MA Foundation for Housing and Community Services. The dimension setting of building and land use cost and residual value are based on the Peng et al. (2011) and Hua et al. (2009) and calculation from this study. The incentive of construction is based on the commonly used principle for splitting the value of real estate development in Taiwan in practice.

5. Results and discussions

In the following section, we explore the annual threshold of rental income for social housing, and how it varies under certainty (calculated using Eq. 3) and uncertainty (calculated using Eq. 10). We also look at the current value of social housing (total building and land use costs) and optimal present value (calculated from Equation 8) as well as the option value of social housing built by the government or private developers in 12 Taipei districts. We then calculated the monthly rent per ping (per square meter) under certainty and uncertainty based on the threshold of income, and compared it with the current market rent values. Songshan and Wanhua District are undergoing preliminary planning for the implementation of short-term policies on social housing. Thus, in the second section, we adopted the conditions of Wanhua District as the basis for sensitivity analysis aimed at elucidating how changes in factors pertaining to the pricing of rent for social housing affects the annual income threshold from social housing under certainty and uncertainty.

5.1 Rent Threshold of Underlying Social Housing, Present Value of Housing, and Option Value of Each District in Taipei

In the case of social housing built by the government (see Table 2), the annual threshold of income under certainty was NTD 106,435,888 in Daan District, NTD 81,408,406 in Zhongzheng District, NTD 71,289,297 in Datong District, and NTD 11,644,532 in Beitou District, which is the lowest. The annual threshold of rental income under uncertainty showed a general increase. Due to excessive annual growth of rent in Nangang District, the model did not obtain the optimal threshold of rent or the present value. Zhongshan District had the highest threshold of rental income under uncertainty (NTD 133,838,843), followed by Zhongzheng District (NTD 127,313,809), and Beitou District (NTD 18,388,016), which was the lowest. These values highlight the variations in the thresholds of rental income when uncertainty is introduced into the model. The greatest difference in the threshold of rental income was NTD 78,494,884 (141.83 %) in Zhongshan District, followed by NTD 45,905,403 (56.39 %) in Zhongzheng District, and 31,571,925 (178.38 %) in Neihu District. The enormous difference between Zhongshan District and Neihu District was due to growth rate of rents and the associated uncertainty.

Table 2: Threshold of rental income, present value, and option value for social housing built by the government in each district in Taipei

District	Threshold of rental income (a=Equation 3)	Optimal threshold of rental income (b=Equation 10)	Difference in threshold of rental income (c=b-a)	Ratio of difference in threshold of rental income to threshold of rental income (d=c/a)	Present value for social housing (e= total construction cost)	Optimal present value for social housing (f= Equation 8)	Option value (g=f-e)	Ratio of option value (h=g/f)
Zhongzheng District	81,408,406	127,313,809	45,905,403	56.39%	3,590,705,908	5,412,993,440	1,822,287,532	33.67%
Datong District	71,289,297	82,063,463	10,774,166	15.11%	2,586,233,852	2,938,012,615	351,778,763	11.97%
Zhongshan District	55,343,959	133,838,843	78,494,884	141.83%	2,658,765,724	6,052,624,045	3,393,858,320	56.07%
Songshan District	65,871,529	85,913,942	20,042,413	30.43%	2,568,745,233	3,272,167,005	703,421,772	21.50%
Daan District	106,435,888	116,190,404	9,754,516	9.16%	4,059,128,365	4,393,934,119	334,805,754	7.62%
Wanhua District	45,745,055	62,857,607	17,112,552	37.41%	1,926,395,028	2,574,967,460	648,572,433	25.19%
Xinyi District	70,303,850	80,155,957	9,852,107	14.01%	2,710,429,277	3,052,275,364	341,846,087	11.20%
Shilin District	14,955,721	19,958,596	5,002,875	33.45%	631,233,077	821,272,894	190,039,817	23.14%
Beitou District	11,644,532	18,388,016	6,743,484	57.91%	523,540,382	796,409,892	272,869,510	34.26%
Neihu District	17,699,347	49,271,273	31,571,925	178.38%	876,077,324	2,282,541,771	1,406,464,447	61.62%
Nangang District	11,940,229	-	-	-	759,113,511	-	-	-
Wenshan District	18,515,899	20,776,744	2,260,845	12.21%	709,294,650	787,240,892	77,946,242	9.90%

Notes: Threshold of rental income (a) is the rent threshold calculated from Equation (3) under certainty; Optimal threshold of rental income (b) is calculated from Equation (10) under uncertainty; Difference in threshold of rental income (c=b-a) is the difference o between the threshold of rental income and the optimal threshold of rental income; Ratio of difference in threshold of rental income to threshold of rental income (d=c/a) is calculated from that difference in threshold of rental income divided by threshold of rental income; The present value of social housing (e) presents the total construction cost; The optimal present value for social housing (f) is calculated from the optimal rental income threshold substituting for Equation (8); The option value (g=f-e) is the optimal present value of social housing minus the present value of social housing; The ratio of option value (h=g/f) is calculated from that option values divided by the optimal present value of social housing.

When we disregarded the effects of uncertainty, the current value of social housing in 11 of the districts ranged from NTD 631,233,077 to NTD 4,059,128,36, based on total building and land use costs. Under uncertainty, the highest present value of social housing was in Zhongshan District (NTD 6,052,624,045), followed by Zhongzheng District (NTD 5,412,993,440), Daan District (NTD 4,393,934,11), and the lowest in Beitou District (NTD 796,409,892). The option values in these districts were NTD 3,393,858,320, NTD 1,822,287,532, NTD 334,805,754, and NTD 272,869,510, respectively. The ratios of the option values in Neihu District, Zhongshan District, and Beitou District were 61.62 %, 56.07 %, and 34.26 % respectively. Daan District presented the lowest ratio at 7.62 %. These findings show that Zhongshan District and Zhongzheng District had the bigger investment value of social housing when rental market uncertainty was taken into account.

When comparing monthly rent per ping (per square meter) and the current market rent under uncertainty (see Table 3), the highest monthly rent per ping was NTD 4,435 (NTD 1,342 per square meter) in Daan District, followed by NTD 3,392 (NTD 1,026 per square meter) in Zhongzheng District, and NTD 2,970 (NTD 899 per square meter) in Datong District. Under uncertainty, the highest monthly rent per ping was NTD 5,577 (NTD 1,687 per square meter) in Zhongshan District, followed by NTD 5,305 (NTD 1,605 per square meter) in Zhongzheng District, NTD 4,841 (NTD 1,464 per square meter) in Daan District, and NTD 766 (NTD 232 per square meter) in Beitou District. According to the ratio of estimated rent to market rent under uncertainty, the highest wa 625.56 % in Zhongzheng District, followed by 621.70 % in Zhongshan District, 553.92 % in Daan District, and 107.31 % in Beitou District. These findings also demonstrate that when uncertainty is taken into account, the rent of social housing per ping in the 11 districts of Taipei was higher than the market rent.

Table 3: Comparison between monthly rent per ping (per square meter) and market rent in each district in Taipei

District	Estimated monthly rent per ping (per ㎡) (a)	Estimated monthly optimal monthly rent per ping (per ㎡) (b)	Difference in rent per ping (per ㎡) (c=b-a)	Ratio of difference in rent to estimated monthly rent (d=c/a)	Market rent per ping (per ㎡) (e)	30% off market rent (per ㎡) (f)	Ratio of estimated monthly rent to market rent (g=a/e)	Ratio of optimal estimated monthly rent to market rent (h=b/e)
Zhongzheng District	3,392 (1,026)	5,305 (1,605)	1,913(579)	56.39%	848 (257)	594 (180)	400.00%	625.56%
Datong District	2,970 (899)	3,419 (1.034)	449(136)	15.11%	660 (200)	462 (140)	450.06%	518.08%
Zhongshan District	2,306 (698)	5,577 (1,687)	3,271(989)	141.83%	897 (271)	628 (190)	257.08%	621.70%
Songshan District	2,745 (830)	3,580 (1,083)	835(253)	30.43%	793 (240)	555 (168)	346.11%	451.42%
Daan District	4,435 (1,342)	4,841 (1,464)	406(123)	9.16%	874 (264)	612 (185)	507.42%	553.92%
Wanhua District	1,906 (577)	2,619 (792)	713(216)	37.41%	680 (206)	476 (144)	280.30%	385.16%
Xinyi District	2,929 (886)	3,340 (1,010)	411(124)	14.01%	759 (230)	531 (161)	385.95%	440.03%
Shilin District	623 (189)	832 (252)	208(63)	33.45%	733 (222)	513 (155)	85.01%	113.45%
Beitou District	485 (147)	766(232)	281(85)	57.91%	714 (216)	500 (151)	67.95%	107.31%
Neihu District	737 (223)	2,053(621)	1,315(398)	178.38%	768 (232)	538 (163)	96.03%	267.31%
Nangang District	498 (150)	-	-	-	853 (258)	597 (181)	58.32%	-
Wenshan District	771 (233)	866(262)	94(28)	12.21%	666 (201)	466 (141)	115.84%	129.98%

Notes: The 1 square meter (㎡) is equal to 0.3025 ping; The monthly rent per ping (a) and the optimal monthly rent per ping (b) are equal to threshold of rental income and optimal threshold of rental income on Table 1 divided by 12 months and 2,000 ping equal the, respectively; The difference in rent per ping (c=b-a) refers to the difference between the monthly rent and the optimal monthly rent. Ratio of difference in rent to monthly rent (d=c/a) is calculated from that difference in rent per ping divided by monthly rent per ping; The market rent per ping (e) is recorded from Tsuei Ma Ma Foundation for Housing and Community Services in 2013; Monthly rent divided by market rent equals the ratio of rent to market rent (g=a/e); The ratio of optimal rent to market rent (h=b/e) is equals to the optimal monthly rent divided by market rent.

When 60 % of the present value of social housing was used by the government to compensate for construction by private developers (see Table 4), the highest threshold of rental income under certainty was NTD 42,574,355 in Daan District, followed by NTD 32,563,362 in Zhongzheng District, and the lowest was NTD 4,657,813 in Beitou Distric. Under uncertainty, the highest optimal rent thresholds were NTD 53,535,537, NTD 50,925,524 and NTD 46,476,161 in Zhongshan District, Zhongzheng District, and Daan District respectively, whereas the lowest was NTD 7,355,207 in Beitou District. Building and land use costs are borne by private developers, such that the present value of social housing under certainty ranged from NTD 96,331,430 in Beitou District to NTD 746,879,619 in Daan District. Under uncertainty, the optimal present value of social housing and the highest option value were respectively NTD 1,032,230,225 and NTD 543,017,331 in Zhongshan District and respectively NTD 952,255,892 and NTD 291,566,005 in Zhongzheng District.

Table 4: Threshold of rental income, present value, and option value for social housing built by the private developer under scenario of 60% of construction incentives

District	Threshold of rental income (a=Equation 3)	Optimal threshold of rental income (b=Equation 10)	Difference in threshold of rental income (c=b-a)	Ratio of difference in threshold of rental income to threshold of rental income (d=c/a)	Present value for social housing (e= total construction cost)	Optimal present value for social housing (f= Equation 8)	Option value (g=f-e)	Ratio of option value (h=g/f)
Zhongzheng District	32,563,362	50,925,524	18,362,161	56.39%	660,689,887	952,255,892	291,566,005	30.62%
Datong District	28,515,719	32,825,385	4,309,666	15.11%	475,867,029	532,151,631	56,284,602	10.58%
Zhongshan District	22,137,584	53,535,537	31,397,954	141.83%	489,212,893	1,032,230,225	543,017,331	52.61%
Songshan District	26,348,611	34,365,577	8,016,965	30.43%	472,649,123	585,196,606	112,547,484	19.23%
Daan District	42,574,355	46,476,161	3,901,806	9.16%	746,879,619	800,448,540	53,568,921	6.69%
Wanhua District	18,298,022	25,143,043	6,845,021	37.41%	354,456,685	458,228,274	103,771,589	22.65%
Xinyi District	28,121,540	32,062,383	3,940,843	14.01%	498,718,987	553,414,361	54,695,374	9.88%
Shilin District	5,982,288	7,983,438	2,001,150	33.45%	116,146,886	146,553,257	30,406,371	20.75%
Beitou District	4,657,813	7,355,207	2,697,394	57.91%	96,331,430	139,990,552	43,659,122	31.19%
Neihu District	7,079,739	19,708,509	12,628,770	178.38%	161,198,228	386,232,539	225,034,312	58.26%
Nangang District	4,776,092	-	-	-	139,676,886	-	-	-
Wenshan District	7,406,359	8,310,697	904,338	12.21%	130,510,216	142,981,614	12,471,399	8.72%

Notes: Threshold of rental income (a) is the rent threshold calculated from Equation (3) under certainty; Optimal threshold of rental income (b) is calculated from Equation (10) under uncertainty; Difference in threshold of rental income (c=b-a) is the difference o between the threshold of rental income and the optimal threshold of rental income; Ratio of difference in threshold of rental income to threshold of rental income (d=c/a) is calculated from that difference in threshold of rental income divided by threshold of rental income; The present value of social housing (e) presents the total construction cost; The optimal present value for social housing (f) is calculated from the optimal rental income threshold substituting for Equation (8); The option value (g=f-e) is the optimal present value of social housing minus the present value of social housing; The ratio of option value (h=g/f) is calculated from that option values divided by the optimal present value of social housing.

In the following, we rank the districts from highest to lowest in terms of the threshold of rental income under certainty: Daan District, Zhongzheng District, Datong District, Xinyi District, Songshan District, Zhongshan District, Wanhua District, Wenshan District, Neihu District, Nangang District, and Beitou District. In the following, we rank the districts from highest to lowest in terms of the optimal threshold of rental income under certainty: Zhongshan District, Zhongzheng District, Daan District, Songshan District, Datong District, Xinyi District, Wanhua District, Neihu District, Wenshan District, Shilin District, and Beitou District.

We obtained the ratio of the difference in threshold of rental income between certainty and uncertainty ranging from 9.16 % in Daan District to 178.38 % in Neihu District. In other words, when uncertainty is taken into account when evaluating the pricing of social housing programs in Taipei, the annual threshold of rental income or monthly optimal rent per ping from from 9.16 % to 178.39 %. The above results indicate that the introduction of uncertainty produces a significant difference in the investment value of social housing and the pricing of rent. Strategies to promote social housing indicate that Daan District is not suitable for social housing because its negative growth rate and lower uncertainty tend to lower the threshold of rental income and investment value under uncertainty. Conversely, Neihu District, Zhongshan District, and Zhongzheng District have higher rent growth rate and uncertainty, which tends to generate a higher threshold of rental income under uncertainty, thereby making these regions suitable for social housing programs.

These findings indicate that a higher ratio of option value is an indication of greater development potential with regard to social housing program. The highest ratio of option value was 61.62 % in Neihu District whereas the lowest was 7.62%
in Daan District. The difference in the ratio of option values under different scenarios indicates that the ratio of option values decreases when social housing is built by private developers. The ratio of option values decreased from 61.62 % to 58.26 % in Neihu District, from 56.07 % to 52.61 % in Zhongshan District, and from 7.62 % to 6.69 % in Daan District. Neihu District has the greatest potential for social housing. However, we found that despite incentives to reduce building and land use costs, the option values of social housing (as evaluated by the government) continues to decline.

The Tsuei Ma Ma Foundation for Housing and Community Services reported only marginal variations in the level of rental markets among the 12 districts of Taipei, there were considerable gaps in the optimal total rental income or rent from social housing per ping under uncertainty. These results also imply that the government should introduce rents of different levels in accordance with the rental market conditions anticipated in the future (Hills, 2007). This also indicates useful strategies for the promotion of policy pertaining to social housing. From the perspective of resource allocation by the government, we recommend that rent discounts or subsidies according to local or regional variations in the housing market. This study suggests that the government adopt higher discounts or subsidy levels to curb higher rent thresholds for a rental market with high uncertainty. For rental markets with lower volatility, a lower discount or subsidy level should be

used to promote policy for fair and economically efficient social housing.
When proposing the rent of social housing based on regional characteristics, the government should reference a reasonable market rent level from steady long-term data. Unfortunately, this type of research has not been conducted for a long time. We recommend that the government compile data pertaining to the leasing market to improve estimates of uncertainty in the rental market.

5.2 Sensitivity Analysis of Parameters Influencing Pricing of Rent for Social Housing

In the following section, we investigate the impact of various changes of parameters influencing rent (including the risk-free rate, growth rate of rent,
volatility, building costs, residuals, and incentive of construction) on threshold of rental income, the optimal threshold of rental income, and option values.

5.2.1 Sensitivity Analysis: Threshold of Rental Income, Optimal Threshold of Rental Income, and Option Value under Various Annual Growth Rate for Rent

We simulated the influence of growth rates of rent on the threshold of rental income, and the optimal threshold of rental income and option value under uncertainty. The simulation scenario was as follows: r =1.21 %, σ =4.610 %, k =1,926,395,028, R =192,639,503, and γ =60 %, (see Figure 1 and Panel A in Table 5). Related studies on real option analysis reported that the influence of rent growth rate on the optimal threshold of rental income should be unpredictable (Peng et al., 2010; Hsieh and Lin, 2016).
Sensitivity analysis of the project in Wanhua District was consistent with these results. When the rent growth rate decreased from 0.045 % to -2 %, the threshold of rental income increased when uncertainty was not introduced into the model. However, the optimal threshold of rental income under uncertainty first decreased and then increased. When the rent growth rate increased to 0.25 %, the threshold of rental income continued to decrease when uncertainty was not taken into account, and the optimal threshold of rental income under uncertainty increased. The increasing annual growth rate of rent has a positive impact on the option value. In other words, when the rent growth rate is higher, the option value increases, and vice versa. This verifies the steadiness of the model, which results in recommendations that are more convincing than those derived using traditional models that fail to take uncertainty into consideration.

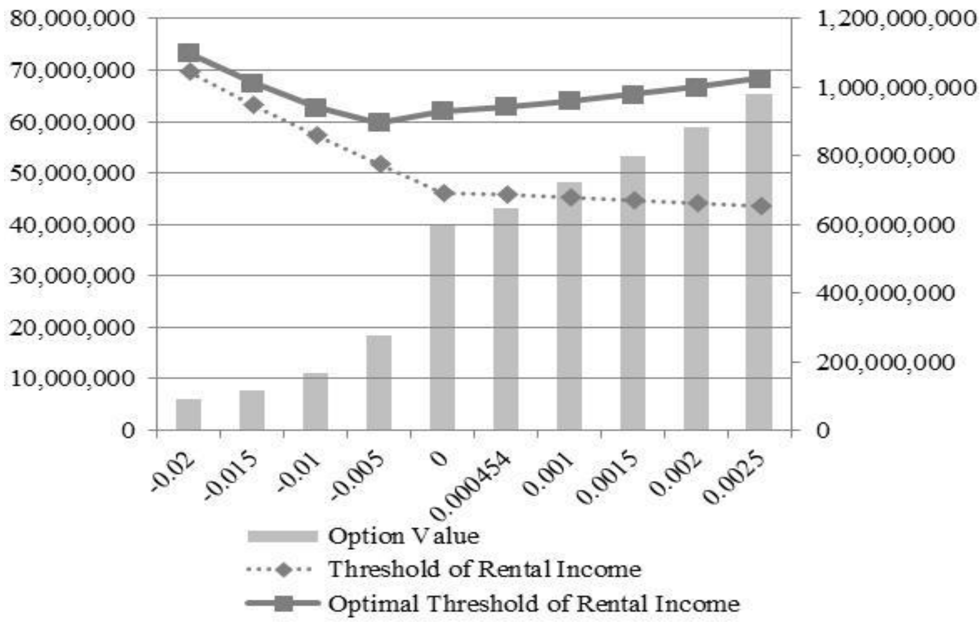

Figure 1: Sensitivity analysis with respect to threshold of rental income, optimal threshold of rental income, and option value under different growth rate for rent

Table 5: Sensitivity analysis of parameter influencing the pricing of rent for social housing

Panel A Sensitivity analysis under different rent growth rate

Growth rate of rate	**-0.020000**	**-0.015000**	**-0.010000**	**-0.005000**	**0.000000**	**0.000454**	**0.001000**	**0.001500**	**0.002000**	**0.0025**
Threshold Rental of Income	69,644,381	63,317,301	57,291,598	51,585,788	46,215,598	45,745,055	45,182,984	44,671,927	44,164,381	43,660,357
Optimal Threshold Rental of Income	73,237,430	67,584,722	62,794,962	59,810,352	62,088,744	62,857,607	63,976,035	65,209,962	66,671,704	68,392,336
Option value	89,446,822	116,850,600	166,542,526	276,420,768	595,473,307	648,572,433	721,124,480	797,098,665	883,567,141	982,108,475

Panel B Sensitivity analysis under different rent uncertainty

Uncertainty	**0.0060**	**0.0160**	**0.0260**	**0.0360**	**0.0461**	**0.0560**	**0.0660**	**0.0760**	**0.0860**	**0.096**
Threshold Rental of Income	45,745,055	45,745,055	45,745,055	45,745,055	45,745,055	45,745,055	45,745,055	45,745,055	45,745,055	45,745,055
Optimal Threshold Rental of Income	48,695,347	51,818,237	55,247,427	58,914,831	62,857,607	66,964,897	71,361,137	76,015,474	80,935,979	86,130,549
Option value	111,817,222	230,175,970	360,143,621	499,139,681	648,572,433	804,240,314	970,859,550	1,147,260,748	1,333,749,826	1,530,626,067

Panel C Sensitivity analysis under different risk-free rate

Risk-free rate	**0.0040**	**0.0060**	**0.0080**	**0.0100**	**0.0121**	**0.0140**	**0.0160**	**0.0180**	**0.0200**	**0.022**
Threshold Rental of Income	37,839,842	39,704,722	41,626,942	43,606,128	45,745,055	47,733,365	49,880,134	52,081,317	54,336,034	56,643,317
Optimal Threshold Rental of Income	68,167,927	63,339,380	61,979,646	62,023,973	62,857,607	64,037,761	65,577,824	67,343,933	69,286,171	71,371,885
Option value	1,389,579,882	1,032,036,406	847,686,895	732,283,320	648,572,433	592,202,918	545,627,201	508,083,256	477,029,352	450,816,419

Panel D Sensitivity analysis under different building and land use cost

Building cost	**20%**	**40%**	**60%**	**80%**	**100%**	**120%**	**140%**	**160%**	**180%**	**200%**
Threshold Rental of Income	9,149,011	18,298,022	27,447,033	36,596,044	45,745,055	54,894,066	64,043,077	73,192,088	82,341,099	91,490,110
Optimal Threshold Rental of Income	12,571,521	25,143,043	37,714,564	50,286,086	62,857,607	75,429,129	88,000,650	100,572,171	113,143,693	125,715,214
Option value	129,714,487	259,428,973	389,143,460	518,857,946	648,572,433	778,286,919	908,001,406	1,037,715,892	1,167,430,379	1,297,144,865

Panel E Sensitivity analysis under different % of residual value

% of residual value	**0%**	**2%**	**4%**	**6%**	**8%**	**10%**	**12%**	**14%**	**16%**	**18%**
Threshold Rental of Income	50,827,839	49,811,282	48,794,725	47,778,169	46,761,612	45,745,055	44,728,498	43,711,941	42,695,385	41,678,828
Optimal Threshold Rental of Income	69,841,786	68,444,950	67,048,114	65,651,279	64,254,443	62,857,607	61,460,771	60,063,936	58,667,100	57,270,264
Option value	720,636,036	706,223,315	691,810,595	677,397,874	662,985,153	648,572,433	634,159,712	619,746,991	605,334,270	590,921,550

Panel E Sensitivity analysis under different % of incentive of construction

% incentive of construction	**0%**	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Threshold Rental of Income	45,745,055	41,170,550	36,596,044	32,021,539	27,447,033	22,872,528	18,298,022	13,723,517	9,149,011	4,574,506
Optimal Threshold Rental of Income	62,857,607	56,571,846	50,286,086	44,000,325	37,714,564	31,428,804	25,143,043	18,857,282	12,571,521	6,285,761
Option value	2,574,967,460	2,103,061,198	1,678,801,495	1,302,188,351	973,221,766	691,901,741	458,228,274	272,201,367	133,821,019	43,087,230

Notes: Based on the Wanhua District, this table depicts the changes of in threshold rental of income, optimal threshold rental of income and option value by simulating different parameters influencing evaluation of social housing, includes rent growth rate, uncertainty, risk-free rate, building cost, of residual value and % of Incentive of construction.

5.2.2 Sensitivity Analysis: Threshold of Rental Income, Optimal Threshold of Rental Income, and Option Value under Different Volatilities

We simulated the influence of rent volatility on the threshold of rental income, and the optimal threshold of rental income and option value under uncertainty. The simulation scenario was as follows: r =1.21 %, δ =4.500 %, k =1,926,395,028, R =192,639,503, and γ =60 % (Figure 2 and Panel B in Table 5). Previous studies predicted that rent volatility would have a positive impact on the optimal threshold of rental income. In other words, they predict that higher volatility would increase the threshold of rental income (Peng et al., 2010; Yao and Pretorius, 2014; Hsieh and Lin, 2016)

Sensitivity analysis indicated no change in the threshold of rental income without uncertainty, due to the fact that traditional evaluation methods do not take volatility into account. However, under uncertainty, an increase in the optimal threshold of rental income increases due to an increase in rent growth rates leads to an increase in the option value. This proves that the pricing model that includes uncertainty results in recommendations that are more convincing than are those based on conventional evaluation models.

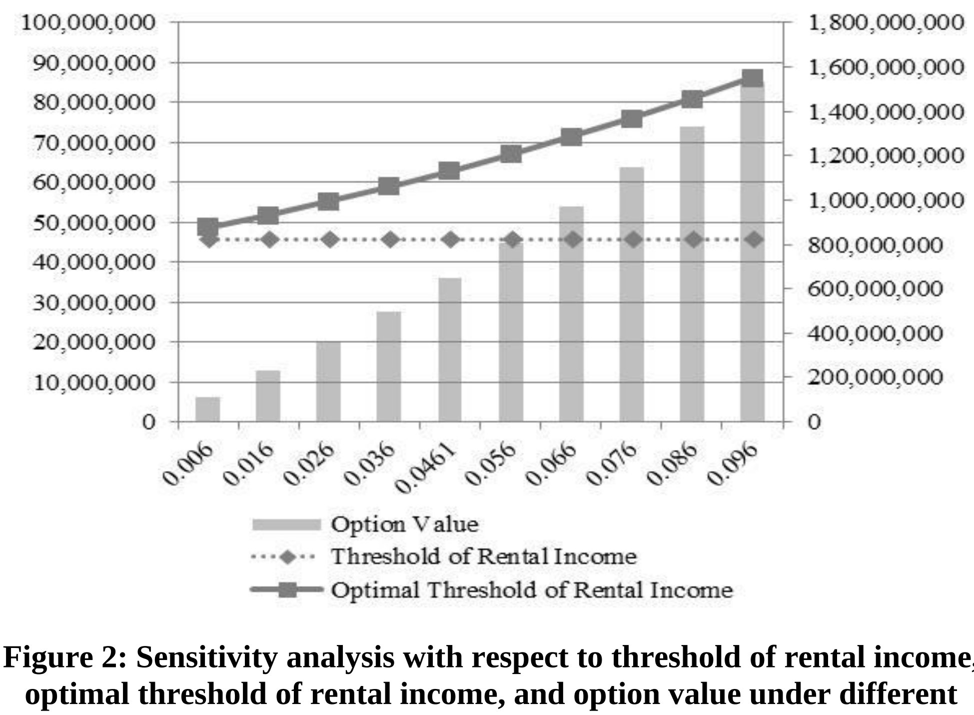

Figure 2: Sensitivity analysis with respect to threshold of rental income, optimal threshold of rental income, and option value under different volatility

5.2.3 Sensitivity Analysis: Threshold of Rental Income, Optimal Threshold of Rental Income, and Option Value under Different Risk-free Rates

We simulated the influence of risk-free rates on the threshold of rental income, and the optimal threshold of rental income and option value under uncertainty (Figure 3 and Panel C in Table 5). The simulation scenario was as follows: δ =4.500 %, σ =4.610 %, k =1,926,395,028, R =192,639,503, and γ =60 %. Previous studies predicted that a risk-free rate can have a positive impact on the optimal threshold of rental income and option values. This means that when a higher risk-free rate leads to a higher threshold of rental income (Chen and Lai, 2013; Yao and Pretorius, 2014). However, other studies have posited that rent growth rates should be taken into account, which means that the risk-free rates do not have a regular impact on the optimal threshold of rental income (Hsieh and Lin, 2016).

Sensitivity analysis revealed that under certainty, changes in risk-free rates have a monotonic positive impact on the threshold of rental income. However, under uncertainty, the optimal threshold of rental income undergoes complicated changes, as indicated in the sensitivity analysis of rent growth rates. In other words, the optimal threshold under uncertainty decreased and then increased. The option values consistently and monotonically decreased as risk-free rates increased. The model results in recommendations that are more convincing than are those based on conventional evaluation models that produce monotonic consistent predictions.

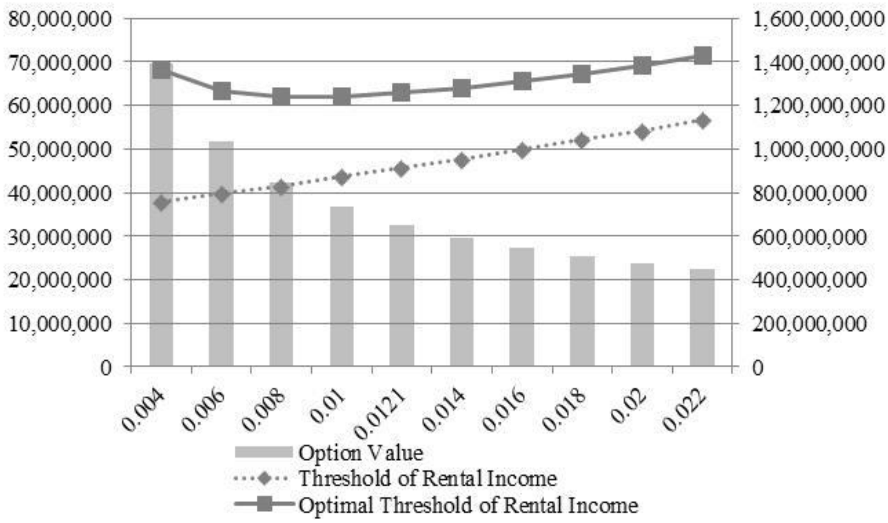

Figure 3: Sensitivity analysis with respect to threshold of rental income, optimal threshold of rental income, and option value under different risk-free rate

5.2.4 Sensitivity Analysis: Threshold of Rental Income, Optimal Threshold of Rental Income, and Option Value under Different Building and Land Use Costs

We simulated the influence of building costs on the threshold of rental income, and the optimal threshold of rental income and option value under uncertainty (see Panel D in Table 5 and Figure 4). The simulation scenario was as follows: r =1.21 %, δ =4.500 %, σ =4.610 %, R =192,639,503, and γ =60 %. Previous studies predicted that the building costs would have a positive impact on the threshold of rental income, in cases where building costs are higher when the threshold of rental income is higher (Peng et al., 2010; Hsieh and Lin, 2016).

The results of sensitivity analysis are consistent with those in previous studies. When the building cost of the project in Wanhua District increased from 20 % to 200 %, and when market uncertainty was considered, the optimal threshold of rental income increased and thewhich led to an increase in the option value increased consequentially. Our results prove the supposition of traditional conventional analysis that the cost of implementing options is higher in areas where it delays the optimal decision-making timing. In other words, when resources are limited, the government should reduce building costs as much as possible in order to meet social housing objectives, including land acquisition and the provision of small amount of loan interest (Hua et al., 2009).

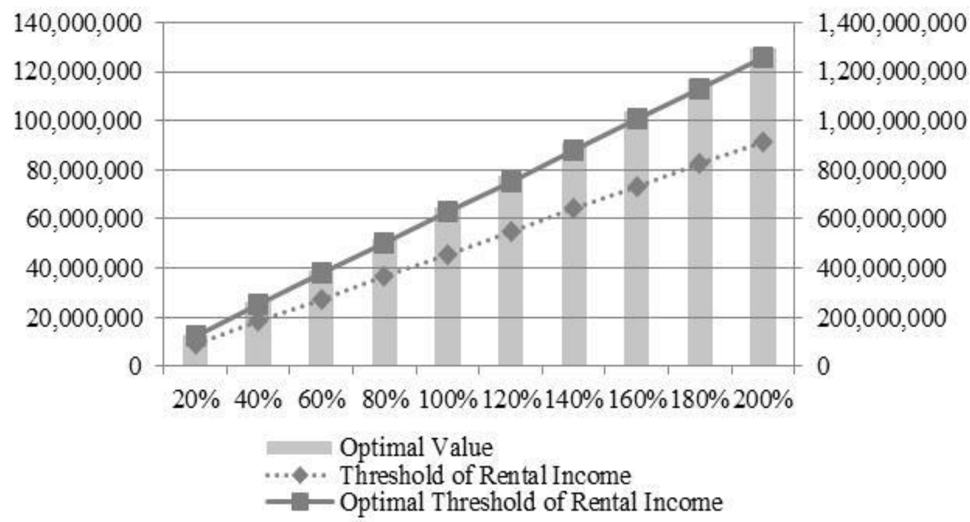

Figure 4: Sensitivity analysis with respect to threshold of rental income, optimal threshold of rental income, and option value under different building and land use cost

5.2.5 Sensitivity Analysis: Threshold of Rental Income, Optimal Threshold of Rental Income, and Option Value under Different Residual Values

We simulated the influence of residual values on the threshold of rental income, and the optimal threshold of rental income and option value under uncertainty (see Panel E in Table 5 and Figure 5). The simulation scenario was as follows: r =1.21 %, δ =4.500 %, σ =4.610 %, k =1,926,395,028, and γ =60 %. The residual values in the model refer to a particular percentage change on building cost, where higher residual values reflect the return on investment by the government. Thus, higher residual values are expected to reduce the threshold of rental income.

Sensitivity analysis revealed that an increase in the %age of residual values led to a decrease in the optimal threshold of rental income and option values. In other words, when the maintenance of social housing is better, the recoverable residual value is higher and the rent of social housing is lower. However, better maintenance requires higher maintenance costs. The government should carefully consider the balance between maintenance, residual values, and rent pricing.

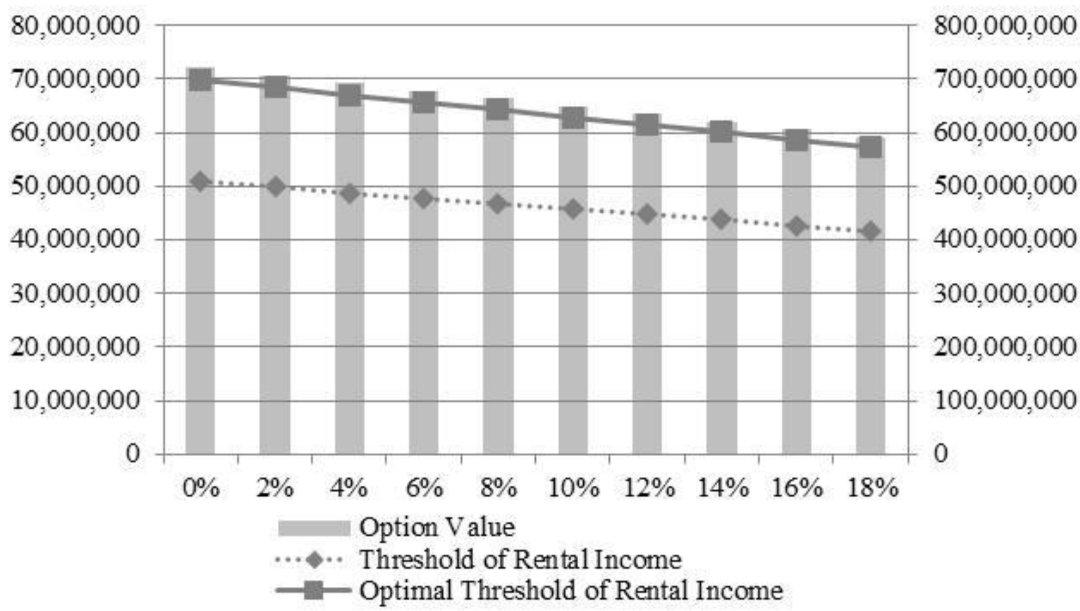

Figure 5: Sensitivity analysis with respect to threshold of rental income, optimal threshold of rental income, and option value under different residual values

5.2.6 Sensitivity Analysis: Threshold of Rental Income, Optimal Threshold of Rental Income, and Option Value under Different Incentives of Construction

We simulated the influence of incentives for construction on the threshold of rental income, and the optimal threshold of rental income and option value under uncertainty (see Panel F in Table 5 and Figure 6). The simulation scenario was as follows: r =1.21 %, δ =4.500 %, σ =4.610 %, k =1,926,395,028, and R =192,639,503. In the model, the government provides a portion of the investment for social housing as an incentive for private developers to undertake construction. We would therefore expect that higher incentives would result in more rental income, and a lower threshold of rental income. This means that the incentives for construction have a negative impact on the threshold of rental income.

Sensitivity analysis revealed when the government provides higher incentives for construction by private developers, the optimal threshold of rental income and option value decrease. The fact that the cost of building social housing is a considerable burden on the government means that government incentives for construction is an important channel by which to fund social housing projects. The balance between building cost and incentives of construction is critical to private participation in social housing, due to the fact that higher building costs or lower incentives for construction both undermine the willingness of private developers to invest. Thus, governments need to carefully consider the means by which to establish appropriate incentives. Allowing appropriately rent increases (Li et al., 2014), designating a portion of the space to commercial use (Li et al., 2016), or establishing tax or loan relief incentives are all the strategies to decrease the burden of related costs incurred by private developers.

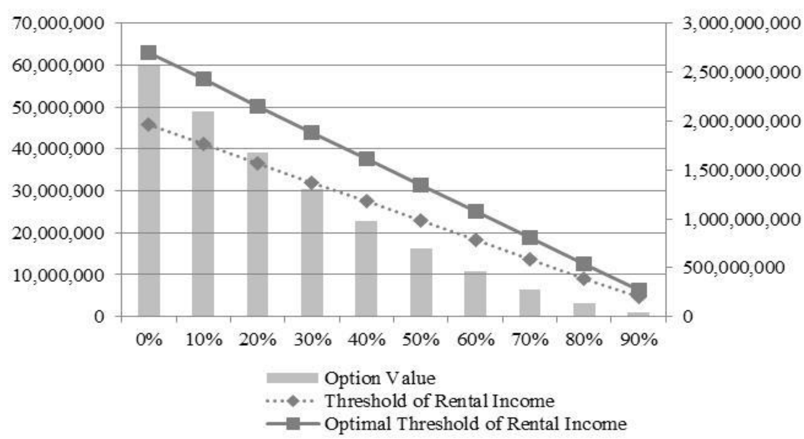

Figure 6: Sensitivity analysis with respect to threshold of rental income, optimal threshold of rental income, and option value under different incentives of construction

6. Conclusions

In a resource-limited environment, the means by which governments promote investments in social housing is challenging challenge, particularly when it comes to determining appropriate rent prices. In Western Europe, social housing is oriented toward self-sufficiency, and countries such as England and the Netherlands have launched reforms in the pricing of rents in order to resolve the question, including the traditional regulation at below-market for rents level and consistent subsidies on households. In this paper, we present a model by which to evaluate social housing based on the principle of self-sufficiency and local housing markets.

Using ROA, the Samuelson–McKean model is was selected to provide a comprehensive model for evaluating and pricing model for social housing based on ROA, in accordance with the principle of self-sufficiency and market mechanisms. This model has been utilized used in published previous studies for evaluating and examining to evaluate the optimal timing of real estate development (Ho et al., 2009; Peng et al., 2010; Hui et al., 2011; Chen and Lai, 2013; Yao and Pretorius, 2014; Wang et al., 2016) and asset pricing (Hsieh and Lin, 2016). Based on this pre-existing model, the study extends the concept to model and to evaluate the feasibility of social housing and pricing rents under uncertainty. According to results of simulation analysis on 12 districts in Taipei, Taiwan, they reveal that threshold of rent and investment values are more convincing under uncertainty is more convincing than under certainty. The sensitivity analysis of the parameters affecting that affect pricing rent prices for social housing is also consistent with findings in the literature, thereby verifying the robustness of this study's the proposed model.

Unlike the models proposed by Ho et al. (2009), Li et al. (2014), and Li et al. (2016), we examined the pricing of rent under uncertainty. Our results indicate that governments should vary rent costs according to market conditions. This would take variability in local or regional housing markets into account when evaluating social housing projects or rent pricing. This model can also market mechanisms in the pricing of rent for social housing (Hills, 2007; Tang, 2008; Tang, 2011; Yuan et al., 2017).

We suggest a number of useful strategies by which to promote social housing. First, from the perspective of resource allocation, rent discounts or subsidy levels should be varied according to the local or regional housing market. Governments could adopt higher discounts or subsidy levels to curb rent thresholds, as in rental markets with greater uncertainty. Likewise, governments should select a lower discount or subsidy level for rental markets with lower volatility, thereby promoting a fair and economically efficient social housing policy. We also provide valuable strategies for the use of vacant housing. When a vacant house is located in a market with higher uncertainty, the government can implement higher subsidies to coax owners to join the rental market. Lower subsidies can be used for vacant houses located in markets with less uncertainty.

In summary, the model presented in this study is responsive to developmental trends in the pricing of social housing, while providing a comprehensive evaluation model with novel methods to promote social housing.

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